Special relativity. Summary

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This note summarises the main elements developed in chapters 2 and 3 of the note entitled: "Another approach to relativity".

Consider two Galilean reference frames Σ (x,y,z,t) and Σ' (x',y',z',t') in rectilinear and uniform motion relative to each other. Each frame of reference has a set of spatial coordinates and synchronised clocks within it, enabling events to be located in space and time.

Let's assume that these frames of reference are in relative motion along a direction taken for the axes of coordinates x and x': let u be the speed of displacement of Σ relative to Σ'.

We retain the principle of relativity: the laws of physics are invariant to any change of Galilean reference frame.

The reference frames are linked by the choice of a common origin (O merged with O') at a common time t = 0. The observation that the clocks between these reference frames are not synchronised is considered to be an experimental fact. It implies that time (date) cannot be considered absolute.

We show that desynchronisation is linked to the existence of a limiting velocity (which turns out to be the free propagation velocity **c** of electromagnetic waves in a vacuum). As a result, the law of composition of velocities (v in Σ and v' in Σ') is written :

$$v' = (v - u) / (1 - \mathbf{uv}/\mathbf{c}^2) \quad (1)$$

Beyond the principle of relativity, the **theory of special relativity** postulates that there is nothing to distinguish one reference frame from the other in terms of the production of an event or carrying out an experiment. This leads to the writing of perfectly symmetrical coordinate change equations, which constitute the Lorentz formulae (avec $\Upsilon = 1/(1- u^2/\mathbf{c}^2)^{1/2}$) :

To go from (x,t) to (x', t'):

$$x' = \Upsilon (x - u\,t) \quad (2)$$

$$t' = \Upsilon (t - u\,x / \mathbf{c}^2)$$

To go from (x',t') to (x, t):

$$x = \Upsilon (x' + u\,t') \quad (2a)$$

$$t = \Upsilon (t' + u\,x'/ \mathbf{c}^2)$$

Equations (2a) are obtained by inverting equations (2), which reflects the assumption that an event cannot be attached to a particular frame of reference.

With these equations, apart from time desynchronization, we observe :

- a dilation of the duration between two events, measured using two different clocks (improper time) compared with the duration measured by a single clock (proper time), i.e. in the frame of reference where these events have the same spatial position:

$$\Delta t' = \Upsilon\, \Delta t \text{ if } \Delta x = 0 \quad \text{and} \quad \Delta t = \Upsilon\, \Delta t' \text{ if } \Delta x' = 0\ ;$$

- a contraction of the lengths measured from the moving frame of reference relative to the lengths in the fixed frame of reference: $\Delta x' = \Delta x / \Upsilon$ if $\Delta t' = 0$ and $\Delta x = \Delta x' / \Upsilon$ if $\Delta t = 0$.

According to the theory of special relativity, the delay between a clock that is set in motion and one that remains stationary is proof of the physical reality of time dilation.

A new approach to relativity

We begin by showing that time dilation leads to a contradiction: the experiment of setting a clock in motion at speed u from the reference frame Σ makes it possible to assert that time passes more slowly in Σ' than in Σ, but the opposite assertion can be made by considering setting a clock in motion at speed $-u$ from Σ'.

This is why we propose an alternative analysis to that of special relativity, based on the following hypotheses:

- Time flows universally (which means that there is no dilation of durations) but the synchronisation of clocks means that it cannot be expressed in the form of absolute time;
- Space is considered to be absolute (no contraction of lengths between reference frames);
- with the hypothesis of the universality of the passage of time, any event appears after a duration T, independent of the reference frame considered, from the instant of origin common to all reference frames.

We will say that an event is "produced" in a reference frame if the time displayed by the clock of the point to which it is attached is identical to this duration ($t = T$). In a frame of reference that is mobile relative to this privileged frame of reference, the event is "perceived" at a different time ($t' \neq T$).[1]

Consider an event produced at point (x,t) in reference frame Σ. In frame of reference Σ', the coordinates (x', t') which satisfy the assumptions set out above are given by the equations below:

$$x' = \Upsilon^2\, (x - u\, t) \qquad (3)$$

$$t' = \Upsilon^2\, (t - u\, x / \mathbf{c}^2)$$

In these equations, the coefficient Υ^2 reflects the fact that, at a given instant in Σ, this reference frame corresponds to a distorted image of Σ' (the clocks appear to be out of sync), which is a kinematic effect.

[1] Unless $x' = 0$, as formulae (2) or (2') show.

For an event produced at the point (x',t') of Σ', the equations are :

$$x = \Upsilon^2 (x' + u t') \qquad (3a)$$
$$t = \Upsilon^2 (t' + u x' / \mathbf{c}^2)$$

Equations (3a) cannot be obtained by inverting equations (3), as the inverse equations are written :[2]

$$x = x' + u t' \qquad (3')$$
$$t = t' + u x' / \mathbf{c}^2$$

Unlike what is accepted by the theory of special relativity, the conjunction of the coordinates x and x' does not constitute a single event, but can be obtained in two different ways:

- considering an event produced in reference frame Σ (considered fixed) at x after a time t: (x,t) corresponds to (x', t'_1);
- considering an event produced in reference frame Σ' (considered fixed) at x' after a time t': (x',t') corresponds to (x, t_1).

Applying equations (3) and (3a) leads to :

$$t_1 = t'_1 = (x - x')/u$$

t_1 and t'_1 therefore represent the time given by the Galilean transformation, but t and t' are different.

Finally, the contradiction inherent in the theory of special relativity lies in the interpretation of the notion of event: if time is not absolute, an event cannot be considered as "produced" in all reference frames, with the exception of the event used to define the common instant of origin.

Approach based on equivalence between mass and energy

We will now show that an approach based on energy considerations provides a physical justification for what has just been presented.

The theory of special relativity introduced the principle of equivalence between mass and energy. We can consider that particle physics has validated this principle. To give it its full generality, we will express it as follows:

"In any galilean reference frame Σ (x, y, z, t), there is the following relationship between the mass m of a particle and its total energy E :

$$E = m\,\mathbf{c}^2 \qquad (4)$$

m depends on the speed of the particle[3] . This is the inertial mass that comes into play in the definition of the particle's momentum."

[2] From equations (3') we can immediately check that the non-dilation of durations and the non-contraction of lengths are satisfied.

[3] We choose not to limit the use of the term mass to rest mass.

The combination of the above principle and the fundamental law of dynamics leads to the relationship giving m as a function of the rest mass :

$$m = \gamma\, m_0 \tag{5}$$

Using the law of composition of velocities (1), we can establish the relationship between the energy of a particle in two reference frames in relative motion (v denotes the velocity of the particle in Σ, v' its velocity in Σ' and u the velocity of Σ' relative to Σ):

$$E' = \gamma\, (m'_0/m_0)\, E\, (1 - u\, v/\mathbf{c}^2) \tag{6}$$

or (inverse relationship): $E = \gamma\, (m_0/m'_0)\, E'\, (1 + u\, v'/\mathbf{c}^2)$ (6')

Relations (6) and (6) show the ratio m'_0 / m_0 . The value to be given to this ratio depends on the choice of a relationship between the particle's rest energies in the two reference frames.

There are two reasonable hypotheses:

- the invariance of mass at rest ;
- that of the conservation of total energy through a change of reference frame.

→ *The first hypothesis* postulates that rest mass is invariant by change of reference frame: this is the postulate of classical Mechanics, but also of the theory of special relativity. We check that this assumption is compatible with Lorentz's formulae (2), (2a).

→ *The second assumption* can be understood through the following example:

Two identical particles of the same rest mass m_0 in Σ are immobile in this frame of reference; they have a velocity equal to - u in Σ' ;

An observer of Σ increases the speed of the first particle from 0 to u by supplying it with energy;

An observer of Σ' reduces the speed of the second particle from - u to 0 by withdrawing energy from it;

The two particles are now at rest with respect to Σ'; what is the rest mass of each in Σ'?

The answer we give is: the first particle has rest mass $m'_0 = \gamma\, m_0$
the second particle has rest mass $m'_0 = m_0 / \gamma$

The total energy acquired by the particle is carried entirely by it and transmitted from one frame of reference to another. Depending on the action exerted, m'_0 can take any value between m_0 / Υ and Υm_0 . We can say that the rest energy of a particle depends on its history.

This second assumption is compatible with the coordinate change equations (3) and (3a). If we carry out an experiment in the Σ reference frame, the transition to the Σ' reference frame is made by adopting $m'_0 = \gamma\, m_0$; we then find equations (3). The transition from Σ' to Σ corresponds to $m_0 = \gamma\, m'_0$ which leads to equations (3a).

The new approach we are proposing therefore involves substituting the invariance of the total energy[4] for the invariance of the rest mass.

An important consequence of the choice of invariance is that it leads to different interpretations for certain phenomena considered as the basis for experimental verification of the theory of special relativity:

- In the RR framework, the delay of moving clocks and the increase in the lifetime of atmospheric muons travelling at high speed are attributed to the dilation of time between reference frames; it is assumed that the clocks continue to deliver the same unit of time;

- with the choice of conservation of total energy by change of reference frame, these phenomena are attributed to the modification of the rest energy of the atoms of the atomic clock or that of the muons, due to the energy received to pass from the reference frame Σ to the reference frame Σ'; the flow of time is not modified, but the clocks no longer deliver the same unit of time.

[4] Note that invariance as we have defined it does not mean that the values given to total energy are always identical in the two reference frames (cf. paragraph 3.3.3 Privileged reference frame, true velocity and true energy of the note "Another approach to relativity".