Gravitational field, Fundamental Principle of Dynamics and Quantum Mechanics

Gravitational field,

8/12/2023

Fundamental principle of dynamics

and Quantum mechanics

Pascal DUBOIS

Keywords : gravitational field; gravitation; gravitational interaction; gravitational energy; fundamental principle; dynamics; quantum mechanics; wave mechanics; gravitational wave; pilot wave; Schwarzschild radius; hypothesis; de Broglie

Summary:

In the note entitled *"Another approach to relativity"* we defined the laws of gravitation based on energy considerations. The purpose of this note is first to present a model of the gravitational field that is consistent with these laws.

We attribute a physical reality to the gravitational field, considered as a distribution of energy throughout space. Gravitational interaction consists of an exchange of energy between gravitational sources and the global field created by these sources.

The gravitational field model includes a mechanism for refreshing the field, constantly adapting it to variations in the energy of its source. This mechanism involves gravitational waves constituting a system that can be considered equivalent to the mass of the source.

The second chapter aims to answer the following question: can we deduce the fundamental principle of dynamics from the properties attributed to the gravitational field?

This question is based on the idea that if a body is at rest in a Galilean frame of reference, it is in equilibrium with its own gravitational field. An impulse applied to this body will set it in motion and increase its energy, which in turn will lead to a change in the gravitational field. And the variation in speed is linked to the variation in the gravitational field. Hence the link between the variation in velocity and the variation in energy, which leads to the fundamental principle of dynamics.

The third chapter seeks to relate the concept of wave-particle duality in quantum mechanics to the concept of the gravitational field.

We show that the characteristics of the gravitational wave that accompanies a moving particle allow it to act as a pilot wave (in reference to the concept introduced by Louis de Broglie's theory) and thus explain the wave behaviour of the particle. This pilot wave is the solution to an equation close to Schrödinger's equation.

Finally, the motion of a particle appears to be entirely controlled by the gravitational field associated with it.

This opens the door to another approach to quantum mechanics.

1. Properties of the gravitational field

1.1. Principles

1.1.1. A new approach to relativity

First of all, it should be remembered that we are dealing with a new approach to special relativity based on the following hypothesis: the **property of matter particles, invariant to a change of Galilean reference frame, is total energy (in the Einsteinian sense: $E = mc^2$) and not rest mass**.

With this hypothesis, distances and durations are conserved in the event of a change of reference frame. The delay of moving clocks is not due to a dilation of time, but to an increase in their energy compared with clocks that remain stationary.

We postulate that the rest mass also varies when the particle undergoes a variation in potential energy in a gravitational field. [1] This explains the gravitational shift in clocks.

It is on the basis of this postulate that new laws of gravitation have been defined that do not require the curvature of space-time.[2] *The principles underlying these laws are described in the appendix.*

1.1.2. Principle of gravitational interaction

In Newtonian gravitational theory, gravitational attraction is modelled using a vector field $\vec{G}(\vec{r})$ proportional to the source mass of the field and varying inversely with the square of the distance. The overall field created by several sources is obtained by vector addition of the individual fields.

Our new approach retains the principle of decay in $1/r^2$, but it is no longer the rest masses of the gravitational sources that are taken into account, but their total energies.

The gravitational field, which is a physical reality, is considered to be a distribution of energy throughout entire space. The gravitational interaction consists of an exchange of energy between gravitational sources and the global field created by these sources. [3]

---

[1] We shall see in section 2.3 that it is possible to justify this hypothesis on the basis of the properties of the gravitational field.

[2] These laws make it possible to predict the results of experiments considered as tests of the theory of general relativity (cf. note entitled *"Another approach to relativity"*). In conclusion, this note explains the similarities and differences with general relativity.

[3] The vector field is no longer used as a principle; however, we will see that the interaction can be simulated by a vector field that differs from the Newtonian field in terms of the associated energy.

1.1.3. Refreshing the gravitational field

We assume that the field is periodically refreshed by two waves travelling at speed c :

- **one propagating energy from the source which it gradually transfers to the field ;**
- **the other propagating the energy it takes from the field back to the source.**

We will refer to these waves as "gravitational waves". [4]

The field thus appears to be the result of these energy movements. It is constantly adjusting to the energy of the gravitational source.

1.2. Gravitational field model

We consider the gravitational field (assumed to be spherical) associated with a mass m at rest in a Galilean reference frame. The energy of the source mass is: $W = m\,\mathbf{c}^2$.

1.2.1. A reminder of the Newtonian model

At a point at a distance r from the source, the gravitational field vector is written as :

$$\vec{G}(\vec{r}) = -\,(Gm/r^2)\,\vec{u} \qquad (1.1)$$

$\vec{u}$ being the unit vector in the radial direction $\vec{r}$.

The force applied to a particle of mass m' subject to gravitational attraction is written :

$$\vec{F} = m'\,\vec{G}(\vec{r}) = -\,(Gmm'/r^2)\,\vec{u} \qquad (1.2)$$

The gravitational vector field is derived from a potential V(r) :

$$\vec{G}(\vec{r}) = -\,\overrightarrow{grad}\,(V) \quad \text{with} \quad V(r) = -\,Gm/r \qquad (1.3)$$

The potential is taken to be zero at infinity and the potential energy is therefore negative.

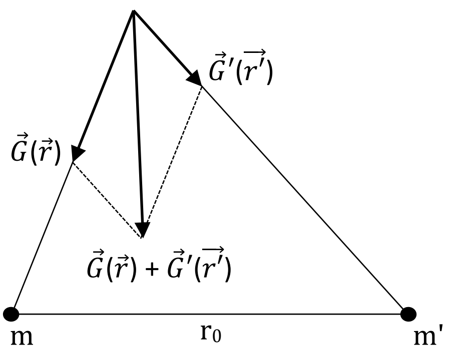
The change in potential energy of the particle of mass m' is :

$$dE_g = (Gmm'/\,r^2)\,dr \qquad (1.4)$$

[4] These waves differ from the gravitational waves of General Relativity.

Field energy

We are going to show that it is possible to introduce into the Newtonian model an interaction energy of the field of two gravitational sources, which can be assimilated to the opposite of the potential energy.



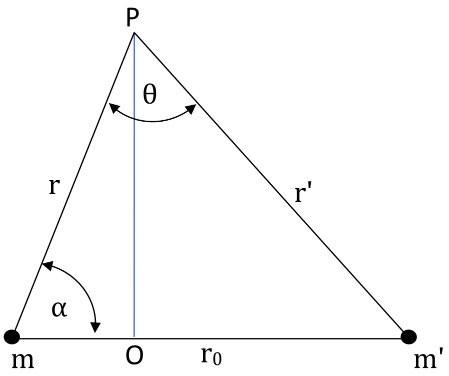
Consider two masses m and m' separated by a distance r_0 . These masses create the gravitational fields $\vec{G}(\vec{r})$ and $\vec{G'}(\vec{r'})$ whose resultant is the vector sum $\vec{G}(\vec{r}) + \vec{G'}(\vec{r'})$.

Let's suppose that, as with the electric field, at each point of the resulting field we can assign an energy density proportional to the square of the norm of the field vector :[5]

$$\delta E(r,r') = k \parallel \vec{G}(\vec{r}) + \vec{G'}(\vec{r'}) \parallel^2 = k \parallel \vec{G}(\vec{r}) \parallel^2 + k \parallel \vec{G'}(\vec{r'}) \parallel^2 + 2k\, \vec{G}(\vec{r})\, \vec{G'}(\vec{r'}) \qquad (1.5)$$

The total energy of the field is obtained by integration over the whole space. Let's look at how this energy varies with r_0 .

The first two terms do not depend on r_0 . Only the third term [$2k\, \vec{G}(\vec{r})\, \vec{G'}(\vec{r'})$] in equation (1.5), which can be thought of as an interaction energy density δE_i , is likely to make a contribution.



Let θ = angle ($\vec{r}$, $\vec{r'}$)

Given relation (1.1) :

$$2k\, \vec{G}(\vec{r})\, \vec{G'}(\vec{r'}) = 2k\, G^2\, mm'\, (\cos\theta / r^2\, r'^2)$$

The two spherical shells centred at m and m', of radius and thickness respectively (r, dr) and (r', dr') have a cylindrical ring in common:

with radius: OP = $r \sin\alpha$ and cross-section: $dr\, dr'/\sin\theta$.

Let's find the amount of interaction energy dE_c contained in the shell (r, dr) by integration with respect to r'.

The integral of $(\cos\theta / r^2\, r'^2)$ is written : $\int (2\pi\, r \sin\alpha \cos\theta / r^2\, r'^2 \sin\theta)\, dr'$

Since $\sin\alpha / r' = \sin\theta / r_0$, it reduces to : $\int (2\pi \cos\theta / r_0\, rr')\, dr'$

[5] cf. for example: L. Brillouin, *L'énigme E = mc² : énergie potentielle et renormalisation de la masse*, Le journal de Physique, Tome 25, 1963.

We know that (relationship in the mPm' triangle): $\cos\theta = (r^2 + r'^2 - r_0^2) / 2rr'$

Finally, the interaction energy dE_c is the sum of two terms:

$$dE_c(r) = (2k\pi G^2 mm'/r_0)(A + B)\, dr$$

with :

$$A = (1/r^2) \int dr'$$

$$B = (1 - r_0^2/r^2) \int (1/r'^2)\, dr'$$

The integration range[6] is: for $r < r_0$: $r_0 - r \le r' \le r_0 + r$

for $r > r_0$: $r - r_0 \le r' \le r + r_0$

The result is as follows:

- for $r < r_0$: $A + B = 0$ In any shell with radius less than r_0, the interaction energy is zero;
- for $r > r_0$: $A + B = 4\, r_0/r^2$ The interaction energy contained in the shell (r, dr) is :

$$dE_c(r) = (2k\pi G^2 mm'/r_0)(4\, r_0/r^2)\, dr = (8k\pi G^2 mm'/r^2)\, dr$$

Integrating with respect to r gives the total interaction energy :

$$E_i = \int_{r0}^{\infty} dEc(r)\, dr = 8k\pi G^2 mm'/r_0$$

The variation of the interaction energy with r_0 is written :

$$dE_i = -(8k\pi G^2 mm'/r_0^2)\, dr_0 \qquad (1.6)$$

This expression can be identified with the opposite of the variation in potential energy (1.4) by choosing :

$$k = 1/8\pi G$$

The total interaction energy is then :

$$E_i = G\, mm'/r_0 \qquad (1.7)$$

This is the energy required to move the sources infinitely apart.

The terms $[k \,\| \vec{G}(\vec{r})| \,\|^2]$ and $k\, [\| \overrightarrow{G'}(\overrightarrow{r'}) \|^2]$ in equation (1.5) are interpreted as the energy of the fields created by the masses m and m', which are assumed to be isolated.

[6] We will see in the next paragraph that the field can only be defined at a minimum distance from the source (it becomes infinite when r tends towards 0).
Strictly speaking, the integration domain must therefore exclude: for r, a ball of radius R_g centred on m; for r', a ball of radius R_g' centred on m'. Here we assume $r_0 \gg R_g$ and R_g'.

1.2.2. New field model

Field energy created by an isolated source

W_g is the total energy of the field and δW_g (r) is its energy density.

The energy of the field decreases in $1/r^2$.

More precisely, the energy contained in a spherical shell of radius r and thickness dr is proportional to W_g/r^2 :

$$dE_c = 4\pi\, r^2\ \delta W_g(r)\, dr = K\, (W_g/r^2)\, dr$$

For the energy of the field to be finite, it must be defined on the space outside a sphere centred on the source; let R_g be the radius of this sphere.

$$\int_{Rg}^{\infty} K\, (W_g/r^2)\, dr = W_g \qquad \text{which means: } K = R_g$$

Therefore :

$$dE_c = (R_g/r^2)\, W_g\, dr \tag{1.8}$$

The energy density is written as :

$$\delta W_g\,(r) = (R_g\,/4\pi\, r^4\,)\, W_g \tag{1.9}$$

Potential energy

The laws of gravitation that we have proposed have been established on the assumption that we can neglect the influence of the particle subjected to the gravitational field on the source of the field. This source is assumed to be fixed in a Galilean frame of reference and to have invariable mass: its field is that of an isolated source.

Referring to the experimental results, we postulated that the variation in potential energy of a particle of energy W' at distance r from the source of the gravitational field is :

$$dE_g = \ 2\ (Gm/\ \mathbf{c}^2 r^2)\ W'(r)\ dr \qquad\qquad 1\ 10)$$

The introduction of the multiplication coefficient 2 with respect to the Newtonian expression is due to the assumption of non-invariance of the rest mass: in our model, half of the variation in potential energy is in fact allocated to the variation in the rest energy of the interacting particle and the other half to the variation in the momentum (see appendix).

The potential is twice the Newtonian potential:

$$V(r) = -\ 2\ Gm/r \tag{1.11}$$

On the other hand, unlike Newtonian theory, the energy W' (which is the total energy) varies with the distance of the particle from the source. Hence the notation W'(r).

From the potential half-energy we can deduce the force applied to the particle to account for the gravitational attraction, which is written :

$$\vec{F} = - (GmW'(r)/\mathbf{c}^2r^2)\,\vec{u}$$

We can therefore still consider that $\vec{F}$ can be deduced from the gravitational field vector :

$$\vec{G}(\vec{r}) = - (GW/\mathbf{c}^2r^2)\,\vec{u} \tag{1.12}$$

This vector is the same as that of the Newtonian field, with the difference that it is the total energy W and not the mass that is taken into account.

Interaction of two sources

Let's repeat the reasoning in paragraph 1.2.1 from equation (1.5) with the field defined by equation (1.12).[7] For the energy to be doubled, the value of the coefficient k must be doubled.

With $k = 1/4\pi G$, the interaction energy becomes (doubling the value given by (1.6)) :

$$dE_i = - 2\,(GW/\,\mathbf{c}^4r^2)\,W'(r)\,dr \tag{1.13}$$

It is the opposite of the potential energy given by equation (1.10).
This relationship is only valid if the effect of the variation in $W'(r)$ *can be neglected in the calculation of* dE_i, *i.e. at great distances from the source.*

We assume that the terms $[k\,\|\,\vec{G}(\vec{r})\,\|^2]$ and $[k\,\|\,\vec{G'}(\vec{r})\,\|^2]$ **can be assimilated to the energy density of the fields created by masses** m **and** m' **considered as isolated**.

Therefore: $\delta W_g(r) = G\,W^2/4\pi\,\mathbf{c}^4\,r^4$

By comparison with the equation (1.9) we obtain the value of R_g :

$$R_g = G\,W^2/\,\mathbf{c}^4W_g$$

If the energy of the field is half the energy of the source ($W_g = W/2$), the value of R_g is :

$$R_g = 2\,G\,W/\,\mathbf{c}^4 = 2\,\mathbf{Gm/c}^2 \tag{1.14}$$

We will see in section 1.3.1 that the field refreshment model we have devised supports the above hypothesis.

R_g **is none other than the Schwarzschild radius. The gravitational field model we have just established therefore shows this radius as the limit below which no gravitational action is exerted.**[8]

[7] In section 1.3.2, we will see that it is possible to explain the interaction between sources on the basis of the field refreshment mechanism, without introducing a vector field.
[8] This is not surprising since, in the comparison made between our approach to gravitation and the theory of general relativity, we refer to the Schwarzschild metric, which is precisely defined outside the sphere of radius R_g.

1.3. Gravitational field refreshment mechanism

1.3.1. Transmitting and receiving energy

In application of the principle adopted in paragraph 1.1.3 we consider, in a reference frame where an isolated gravitational source is at rest, two spherical waves of velocity equal to $\mathbf{c}$, one propagating energy towards the field (outgoing wave), the other propagating energy towards the source (incoming wave).

We also assume that the transfer and withdrawal of energy from the field take place alternately for durations equal to a period T.

As a result, in each spherical shell of thickness cT, the field energy fluctuates between the energy supplied by the outgoing wave and a zero value. We will assume that the average total energy of the field is equal to half the energy emitted or received at the Schwarzschild limit.

In paragraph 1.2.2 we made the assumption that the energy of the field is equal to half the energy of the source. **The energy of the outgoing or incoming gravitational waves is therefore equal to the energy of the source.**

Let us now assume that: $\mathrm{cT} = \mathrm{R_g}$ (so $\mathrm{T} = \mathrm{R_g}\,\mathbf{/c = }\,2\,\mathbf{Gm/c}^3$).[9] (1.15)

The diagram below illustrates the oscillation of the gravitational field created by the alternating transfer and extraction of energy in spherical shells of thickness $\mathrm{R_g}$:

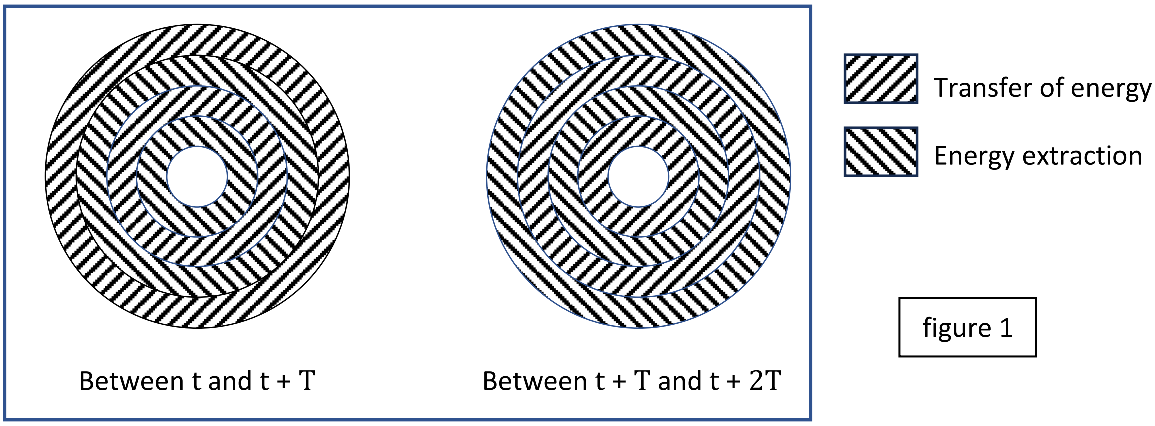


figure 1

Remember that the energy of the field decreases in $1/\mathrm{r}^2$ and that the energy transported by gravitational waves varies in $1/\mathrm{r}$. In a reference frame where the source is at rest, the conjugation of these waves does not displace any energy, but simply causes an oscillation of the field.

The gravitational wave system can be considered as equivalent to the mass of the source.[10]

---

[9] We shall see in Chapter 2 that this choice makes it possible to deduce the fundamental principle of dynamics from this property of the gravitational field.

[10] On a different basis, G. La Frenière expressed the idea that matter is made up of waves: http://web.archive.org/web/20110901223405/http://glafreniere.com/matter.htm

1.3.2. Interacting sources: energy exchange between sources and field

We are now able to propose a mechanism for the exchange of energy between sources and the gravitational field that explains the potential energy.

We consider two sources of mass m and m' separated by a distance r_0.

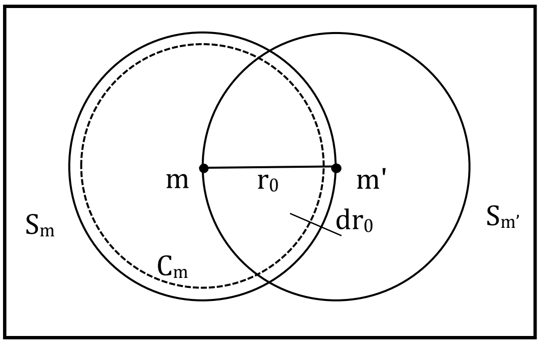


figure 2

In section 1.2.1 we saw that the interaction energy is zero inside the spheres S_m and $S_{m'}$ of radius r_0. This means that :

- the source m' does not interfere with the field of m inside the sphere S_m ;
- the source m does not interfere with the field of m' inside the sphere $S_{m'}$;

Let's assume that the sources of masses m and m' approach each other by a distance $|dr_0|$.

The variation in interaction energy is due to the withdrawal of energy:

- of the source m' on the field, created by m, contained in the shell C_m of centre m, radius r_0 and thickness $|dr_0|$; the energy of this shell is: $(R_g/r_0^2)\ W\ |dr_0|$; [11]

- of the source m on the field, created by m', contained in the shell $C_{m'}$ of centre m', radius r_0 and thickness $|dr_0|$; the energy of this shell is: $(R_g'/r_0^2)\ W'\ |dr_0|$.

For the source m, energy is drawn off over a time T proportional to the energy W; similarly, for the mass m', energy is drawn off over a time T' proportional to W'. We'll assume that energy is withdrawn in proportion to the withdrawal times :

$$W/(W + W') \text{ and } W'/(W + W').$$

The energies extracted are therefore worth :

- for source m': $(W'/(W+W'))(R_g/r_0^2)\ W\ |dr_0| = 2\ G\ (W^2W'/(W+W'))/\mathbf{c}^4r_0^2\ |dr_0|$ (1.16)

- for source m: $(W/(W+W'))(R_g'/r_0^2)\ W'\ |dr_0| = 2\ G\ (W'^2W/(W+W'))/\mathbf{c}^4r_0^2\ |dr_0|$

The continuous refreshing of the fields means that their energy is constantly being restored. This explains why the energy drawn by one source from the field of the other can be considered as interaction energy added to the energies of the fields of the isolated sources.

[11] To determine the energy drawn, the maximum value (W) of the field energy must be taken into account.

The variation in interaction energy is the sum of the two terms above, i.e. :[12]

$$dE_i = - 2\ (G\ WW'/\mathbf{c}^4 r_0^2)\ dr_0\ = - 2\ (G\ mm'/r_0^2)\ dr_0$$

In our case (m >> m'), the interaction energy is entirely captured by m'. This one is the opposite of the variation in potential energy of this particle (see equation (1.10)).

The method can be extended to the interaction of any number of sources.

1.3.3. Photons in a gravitational field

We admitted that the phenomenon of energy exchange between a gravitational field and a particle of non-zero mass could be transposed to photons.[13] How can this be explained?

We assume that electromagnetic waves have the same capacity to exchange energy with the gravitational field as gravitational waves.

A photon of energy E_φ , which can be considered as a fragment of an electromagnetic wave, therefore also has this capacity. Assuming that this photon belongs to a spherical electromagnetic wave of total energy $N\ E_\varphi$, equation (1.15) in the previous paragraph becomes :

$$N|d\ E_\varphi\ | = (\ NE_\varphi\ /(W+N\ E_\varphi))(R_g/r_0^2)\ W\ |dr_0|$$

Therefore: $|d\ E_\varphi\ | = (E_\varphi\ /(W+N\ E_\varphi))(R_g/r_0^2)\ W\ |dr_0| \approx E_\varphi\ (R_g/r_0^2)\ |dr_0|$ (if $W \gg N\ E_\varphi$) (1.16)

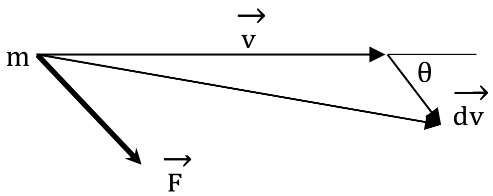
which is the equation giving the variation in photon energy.

[12] Of course, we find the value given by equation (1.13), taken from the energy density of the mean field.
[13] cf. note entitled *"Another approach to relativity"* paragraph 5.1.3.

2. Gravitational field and the Fundamental Principle of Dynamics

2.1. Fundamental equation of dynamics

Consider a body of mass m[14] moving at speed $\vec{v}$. This body is subjected to the action of a force $\vec{F}$.



The fundamental equation of dynamics is written :

$$d(m\,\vec{v})/\,dt = \vec{F} \qquad (2.1)$$

The change in energy is equal to the work of the force :

$$dW = \vec{F}\,(\vec{v}\ dt)$$

Equation (2.1) can therefore be written as :

$$\vec{v}\ d(m\,\vec{v}) = dW \qquad (2.2)$$

Let's say: $dv = d(\|\vec{v}\|)$. Given the principle of equivalence between mass and energy ($W = m\,\mathbf{c}^2$) and the fact that $\vec{v}\ d\,\vec{v} = v\,dv$ (because $dv \approx \|\, d\,\vec{v}\,\| \cos\theta$), we have :

$$(v\,\mathbf{dv/c}^2)\,W + (v^2\,/\mathbf{c}^2)\,dW = dW$$

or :

$$dv = (1 - v^2\,/\mathbf{c}^2)\,(\mathbf{c}^2\,/v)\,dW/W \qquad (2.3)$$

We will show that it is possible to obtain the latter equation from the properties of the gravitational field described in Chapter 1.

2.2. Establishing the fundamental equation of dynamics from the field

Consider a body of mass m and total energy W with velocity v in reference frame Σ. This body undergoes an impulse transferring to it an increment of energy ΔW during the period T that punctuates the field adjustment (cf. paragraph 1.3.1). We are looking for its velocity increment Δv.

In the reference frame Σ' where the body is at rest, the Schwarzschild boundary consists of a sphere of radius R_g .

We have seen that R_g is proportional to W (equation (1.13)). The energy increment ΔW therefore increases the Schwarzschild radius by an amount ΔR_g such that :

$$\Delta R_g\,/R_g = \Delta W/W \qquad (2.4)$$

[14] In this chapter, we use the term "body" to refer to gravitational field sources because the reasoning applies whatever the size of these bodies. It should also be remembered that the term "mass" is not reserved for rest mass.

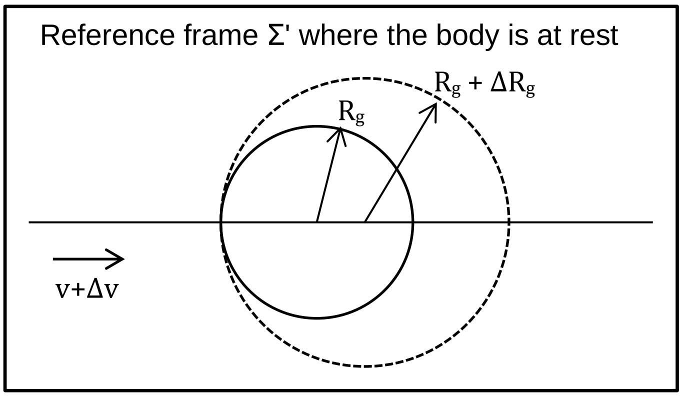


figure 3

We can assume that the increase in radius is accompanied by a displacement of the sphere, as shown in Figure 3: the sphere of radius $R_g + \Delta R_g$ tangents the sphere of radius R_g . The centre of the sphere then moves by the amount ΔR_g. This is equivalent to considering that the body is limited in its displacement by the impossibility of interacting with its own field. All we need to do is determine the duration associated with the displacement ΔR_g to deduce the additional velocity of the body.

The period T corresponds in Σ to a displacement of the centre of the field from O to O_1 such that:

$OO_1 = v\,T$.

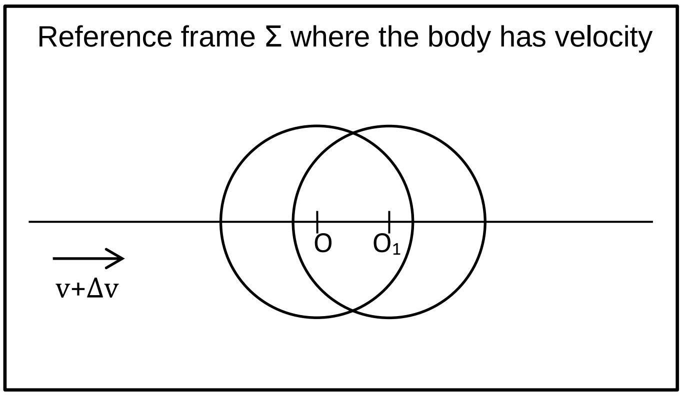


figure 4

Since we assume that the field is established at speed $\mathbf{c}$, the adjustment over a distance equal to OO_1 takes place in a time :

$$\Delta T = v\, \mathbf{T/c}.$$

It is during this time that the radius is adjusted to its new value when an impulse is given.

We have chosen: $T = R_g\, \mathbf{/c}$ (see equation (1.14)).

Therefore : $\Delta T = v\, R_g\, \mathbf{/c}^2$

The speed increment in reference frame Σ' is :

$$\Delta u = dR_g\ /\Delta T = (\mathbf{c}^2 /v)\ \Delta R_g /R_g \tag{2.5}$$

The law of composition of velocities leads to the following increment in reference frame Σ :

$$\Delta v = (1 - v^2 /\mathbf{c}^2)\, \Delta u = (1 - v^2 /\mathbf{c}^2)\, (\mathbf{c}^2 /v)\, \Delta R_g /R_g$$

or :

$$\Delta v = (1- v^2 /\mathbf{c}^2)\, (\mathbf{c}^2 /v)\, \Delta W/W \qquad (2.6)$$

which is the desired result.

What happens if the body of mass m *cannot be considered as a single gravitational source?*

The above reasoning applies to each mass fraction m_e source of a field defined from a single central point. The result (2.1) is broadly the same if, for any elementary mass, the velocity is identical (= v) and the ratio $\Delta W_e /W_e$ identical (= $\Delta W/W$). In other words, in the case of a rigid body.

Note: the hypothesis of the existence of black hole implies that masses with distinct gravitational fields can merge into a single source.

2.3. Sharing potential energy in our approach to gravitation

From the reasoning in paragraph 2.2 it is possible to justify the third principle *in our new approach to gravitation:*

"The rest energy of a particle of non-zero mass varies with its distance from the source. In the absence of external forces, the variation in energy associated with the rest energy and the variation in energy associated with the momentum are each equal and opposite to half the variation in potential energy".

The gravitational energy received by the source is related to a duration of $2T$. Equation (2.2) therefore becomes :

$$\Delta T = 2\, v\, R_g /\mathbf{c}^2$$

And equation (2.3) becomes :

$$v\, dv/\mathbf{c}^2 /(1- v^2 /\mathbf{c}^2) = (dW/2)/W$$

We can assume that the change in speed is related to half of the change in energy, and that the other half must therefore be allocated to the change in energy related to the rest mass.

3. *Gravitational field and quantum mechanics*

3.1. Wave behaviour

3.1.1. Louis de Broglie's hypothesis

An electromagnetic wave of frequency ν can be considered to be made up of photons (quanta of energy $\mathbf{h}\,\nu$), emitted at frequency ν.

Conversely, even an isolated photon of energy E_φ can be associated with a wave of frequency $E_\varphi\,/\mathbf{h}$, which allows us to account for the wave-like behaviour of the photon.

Louis de Broglie extended this wave behaviour to particles of non-zero mass: to a particle at rest of mass m_0 , he associated a periodic phenomenon of frequency ν_0 such that :

$$m_0\,\mathbf{c}^2 = \mathbf{h}\,\nu_0 \tag{3.1}$$

Considering that this equation must be invariant by the Lorentz transformation, he defines this periodic phenomenon as a wave whose characteristics are given in the table below (in comparison with those of photons):

	photon	Material particle (mass m, speed v)
energy	$E = \mathbf{h}\,\nu$	$E = \mathbf{h}\,\nu = m\,\mathbf{c}^2$
momentum	$p = E/\mathbf{c}$	$p = m\,v = E\,v/\,\mathbf{c}^2$
wavelength	$\lambda = \mathbf{h}/p = c/\nu$	$\lambda = \mathbf{h}/p = (\mathbf{c}^2\,/v)/\nu$

A particle moving at speed v can thus be associated with a wave having frequency $\nu = m\,\mathbf{c}^2\,/\mathbf{h}$ [15] , speed $V = \mathbf{c}^2\,/v$ and wavelength :

$$\lambda = \mathbf{h}/mv \tag{3.2}$$

With a speed greater than $\mathbf{c}$, this wave cannot carry any energy. De Broglie calls it a "phase wave" associated with the propagation of the particle because, for a fixed observer, there is agreement between the phase of the wave and the phase of a clock that has the frequency ν_0 in the reference frame linked to the particle.[16]

The phase wave explains the wave-like behaviour of material particles. The relationship (3.2) has been verified by several diffraction experiments on electrons, but also on more massive particles.

[15] Remember that m is the mass corresponding to the total energy of the particle. For us, m is therefore also the rest mass in the frame of reference where the particle is stationary. De Broglie writes: $\nu = \gamma\,m_0\,\mathbf{c}^2\,/\mathbf{h}$.
[16] *Recherches sur la théorie des quanta*, doctoral thesis defended in Paris on 25 November 1924.

3.1.2. Double solution theory and pilot wave

Louis de Broglie presents his conception of wave mechanics as follows:

"When, in 1923-1924, I developed the results that appear in my doctoral thesis, I started from the idea that it was necessary to extend to all particles the coexistence of waves and particles discovered in 1905 by Einstein for light in his theory of light quanta (photons). To achieve this, I felt it was essential to adopt the physical image of a wave, undoubtedly of very low intensity, which would transport particles conceived as large local concentrations of energy incorporated into the wave". [17]

On this basis, in May 1927 he proposed the "double solution theory". [18] This theory led to the idea that the wave associated with the particle guided its movement: it was a pilot wave.

Given the purely probabilistic development of the interpretation of quantum mechanics, the idea of a pilot wave having a physical reality was put aside. It was not revived until after 1950 and developed by David Bohm.[19]

We will now define the characteristics of gravitational waves and show that these characteristics enable them to act as pilot waves.

3.2. Characteristics of gravitational waves

3.2.1. Quantum of gravitational energy

In Chapter 1 we associated a period $T = R_g / \mathbf{c}$ with gravitational waves, which controls the refreshment and oscillation of the gravitational field. We also saw that these waves carry (in the Schwarzschild limit) an energy equal to the total energy W of the source.

We postulate that the energy emitted or received each time the field is refreshed constitutes an indivisible quantum. However, as the energy transported varies in $1/r$, the energy of the quantum varies with the distance from the source:

$$W_q (r) = (R_g / r)\, W$$

By extending the Planck-Einstein relation for photons, we postulate that any quantum of gravitational energy W can be associated with a wave fragment whose frequency ν_S verifies the relation:[20]

$$W = \mathbf{h}\, \nu_S \tag{3.3}$$

[17] *Louis de Broglie, sa conception du monde physique,* Gauthier-Villars Ed, 1973

[18] Louis de Broglie, *La mécanique ondulatoire et la structure atomique de la matière et du rayonnement*, Le journal de Physique et le radium, May 1927.

[19] D. Bohm & B.J. Hiley, *The undivided universe*, Routledge Ed, 1993.

[20] Note that the frequency is associated with the quantum of energy of the gravitational wave and not directly with the energy of the particle.

The gravitational wave can be seen as a succession of energy quanta $\mathbf{h}\nu_S$ (originally) emitted (or received) at the frequency $1/2T$. It **is the frequency ν_S, and not the emission frequency, that controls the wave behaviour of the quanta.**

The analogy between photons and gravitational energy quanta cannot be extended to a complete analogy between electromagnetic waves and gravitational waves because, for the latter, the frequency linked to the energy of the quanta (ν_S) and the emission frequency ($1/2T$) are distinct:

ν_S is proportional to m, $1/2T$ is proportional to $1/m$. [21]

Note: for an electromagnetic wave moving photons of constant energy, the power varies as the square of the frequency[22] , whereas for a gravitational wave, it is the energy of the quantum that varies as the square of the frequency (at a given distance r).

Wave equations

Let's assume that, in reference frame Σ' where the source is at rest, we can associate a sinusoidal wave function with pulsation :

$$\omega_S = 2\pi\,\nu_S$$

The propagation equations are as follows:

- for the outgoing wave : $U_s(\vec{r'},t') = A(r')\sin(\omega_S(t' - r'/\mathbf{c}))$ (3.4)
- for the incoming wave : $U_e(\vec{r'},t') = -A(r')\sin(\omega_S(t' + r'/\mathbf{c} - 4R_g/\mathbf{c}))$

We have : $U_s(R_g, R_g/\mathbf{c}) = 0$ et $U_e(3R_g, R_g/\mathbf{c}) = 0$,

which ensures the correct phase shift between incoming and outgoing waves.

What do U_s and U_e represent?

In view of the remark made above and by analogy with the Poynting vector, which gives the power per unit area of an electromagnetic wave, let's assume the following relationship between the energy of the gravitational quantum per unit area and U_s (or U_e):

$$W_q(r') / 4\pi r'^2 = k\,\mathrm{avg}(U_s^2) \qquad (3.5)$$

avg( ) representing the time average over a period equal to the wave period.

Therefore : $\mathrm{avg}(U_s^2) = A(r')^2/2$

To express the interaction energy of the gravitational field (see section 1.2.2 Interaction of two sources), we used $k = 1/4\pi G$.
To take into account the energy dissipation of the wave, this value must be multiplied by $1/r'$:

$$k = 1/4\pi G r'$$

---

[21] We will come back to this difference in paragraph 3.3.4.
[22] cf. note entitled *"Another approach to relativity"*, paragraph 4.2.2.

With this value of k, the following expression for $A(r')^2$ can be derived from equation (3.5) :

$$A(r')^2 = 4\, G^2\, m^2\, / r'^2$$

$A(r')$ **can therefore be identified with the potential** (see equation (3.11)):

$$A(r') = -\,2\, Gm/r' \qquad (3.6)$$

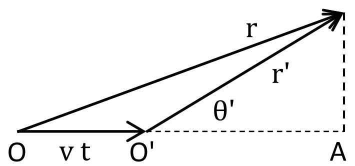
3.2.2. Source in motion

Let's now consider that the particle of mass m, the source of the gravitational field, is moving rectilinearly and uniformly at speed v in reference frame Σ (x,y,z,t). We choose the x axis parallel to the velocity.

The conjugation of the incoming and outgoing waves leads to the equation :

$$U(\vec{r'}, t') = -\,2\, A(r') \sin(\omega_S\, (r'/\mathbf{c} + 2R_g\, /\mathbf{c})) \cos\, (\omega_S\ \ (t' - 2R_g\, /\mathbf{c})) \qquad (3.7)$$

which is the standing wave equation.



OA = x

O'A = x'

The equations for the change of reference frame between Σ' and Σ can be written as[23] :

$$x' = x - v\, t$$
$$t' = t - v\, \mathbf{x}/\mathbf{c}^2$$

Where θ' is the angle of the ray $\vec{r'}$ with respect to $\vec{v}$:

$$r' = x'\, /\cos\theta' = (x - v\, t)/\cos\theta'$$

And : $\vec{r'} = \vec{r} - \vec{v}\,t$

In the reference frame Σ, the equation (3.7) for the resultant wave therefore becomes :

$$U(x, \theta', t) = -\,2\, A(r') \sin\, (\omega_S\, ((x - v\, t)\, /\mathbf{c} \cos\theta' + 2R_g/\mathbf{c})) \cos\, (\omega_S\, (t - v\, x/\mathbf{c}^2 - 2R_g/\mathbf{c})) \qquad (3.8)$$

$$U(x, \theta', t) = 2\, A(r') \sin\, ((v\, /\mathbf{c} \cos\theta')\ \omega_S\, (t - x/v - 2R_g \cos\theta'/v))\cos(\omega_S\, (\, t - x/(\, \mathbf{c}^2/v) - 2R_g/\mathbf{c}))$$

or as a function of $\vec{r}$:

$$U(\vec{r}, t)\ \ = -\,2\, A(\|\,\vec{r} - \vec{v}t\|) \sin\, (\omega_S\, (\,\|\,\vec{r} - \vec{v}t\|/\,\mathbf{c} + 2R_g/\mathbf{c})) \cos\, (\omega_S\, (\,t - \vec{v}.\vec{r}\,/\,\mathbf{c}^2 - 2R_g/\mathbf{c})) \qquad (3.8')$$

This equation combines a group wave and a phase wave, which can be considered as plane waves propagating in the direction of the particle's velocity:

[23] cf. note *"Another approach to relativity"*, § 2.4. The basic reference frame is Σ' and Σ moves at speed - v relative to it.

- the group wave $[2\,A(r')\sin((v/\mathbf{c}\cos\theta')\,\omega_S\,(t - x/v - 2R_g\cos\theta'/v))]$ has a frequency $[(v/\mathbf{c}|\cos\theta'|)\nu_S]$ which varies with the direction of radiation considered (in Σ'). Its speed is v, like the particle whose motion it accompanies; it is this wave that displaces the energy;

- the phase wave $[\cos(\omega_S\,(t - x/(\mathbf{c}^2/v) - 2R_g/\mathbf{c}))]$ has the same frequency as the gravitational waves in Σ': ν_S , and its speed is equal to $\mathbf{c}^2/v$.

We can see straight away that the phase wave fulfils the conditions of De Broglie's hypothesis:

- its frequency is: $\nu_S = \mathbf{mc}^2/h$
- its speed is: $\mathbf{c}^2/v$.

Note: The wavelength of gravitational waves is :

$$\lambda_S = \mathbf{c}/\nu_S = \mathbf{h}/mc \tag{3.9}$$

This is the Compton wavelength, which is also the wavelength of the group wave for θ' = 0.

3.2.3. Displaced energy

In the reference frame Σ where the source is in motion, the resulting wave displaces in a time Δt an energy equal[24] to that contained in an infinite plane slice perpendicular to v and of thickness v Δt (as shown below on the assumption that: $v\,\Delta t < R_g$).

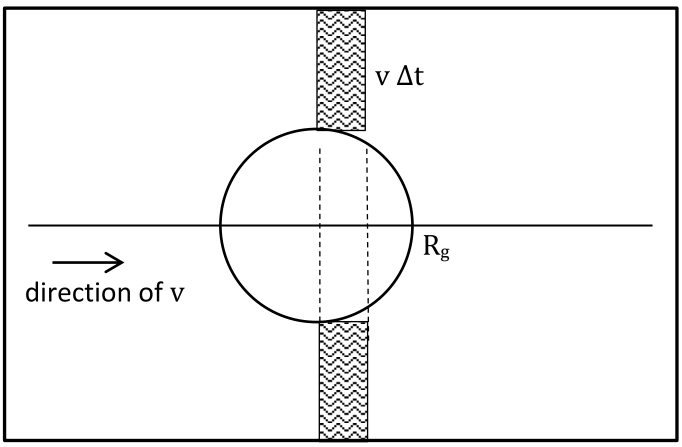
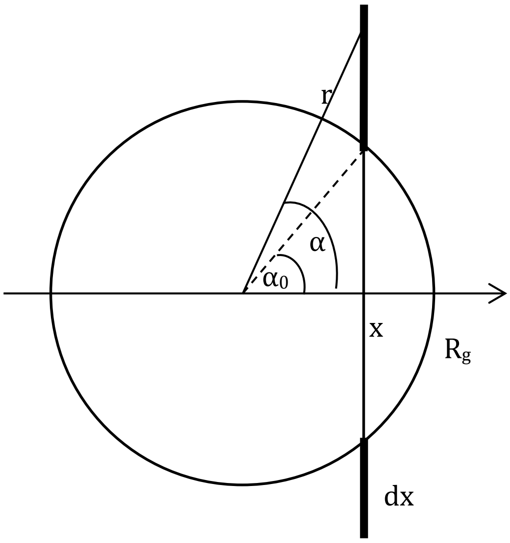


figure 6

Let us first calculate the energy $dW_g(x)$ contained in a flat slice of thickness dx of the gravitational field perpendicular to the direction of motion, at a distance x from the centre of the source less than the Schwarzschild radius $R_g = 2\,Gm/\mathbf{c}^2$.

[24] Remember that the average energy of the field is equal to half that of the source and is independent of the Galilean reference frame considered.



The energy density of the field has the value (see (1.9)):

$\delta W_g(r) = (R_g/4\pi\, r^4)\, W_g = G\, m^2/(4\pi\, r^4)$

$dW(x) = dx \int_{\alpha 0}^{\Pi/2} (G\, m^2/\,(4\pi\, r^4))\, (2\pi\, x\, tg\, \alpha)\, x\, d(tg\, \alpha)$

Since $r = x/\cos\alpha$:

$dW(x) = -\, dx\, (Gm^2/2x^2) \int_{\alpha 0}^{\Pi/2} \sin\alpha \cos\alpha\, d\alpha$

$dW(x) = -\, dx\, (Gm^2/4\, x^2)\, (\sin^2(\pi/2) - \sin^2 \alpha_0)$

Since $R_g = x/\cos\alpha_0$:

$$dW(x) = dx\, (Gm^2/4\, R_g^2) = dx\, (\mathbf{c}^4/16G) \qquad (3.10)$$

Note: as long as the slice intercepts the Schwarzschild limit, the energy contained in the slice is proportional to its thickness and independent of the source mass of the gravitational field. However, for a thickness $\Delta x = R_g$, the energy is proportional to the mass, as expected.

For a displacement time equal to 2T, the displacement is : $\Delta x = 2v\, T = 2v\, R_g/c$

(3.10) → $$\Delta W_g = (2v/\mathbf{c})\, (Gm^2/4R_g) = m\, \mathbf{c}\, v/4 \qquad (3.11)$$

It is easy to check that this result is to be multiplied by R_g^2/x^2 if we place ourselves at a distance $x > R_g$

So, for a period equal to 2T :

- gravitational waves emit and extract energy equal to $m\, \mathbf{c}^2$;
- at distance x in reference frame Σ, everything happens as if the resulting wave displaced an energy equal to $(R_g/x)^2\; m\, \mathbf{c}\, v/4$.

3.3. Resulting gravitational wave as a pilot wave

3.3.1. Particle trajectory

We have seen that the resulting wave satisfies the de Broglie hypothesis. Can it act as a pilot wave to define the particle's trajectory?

Consider the phase factor:

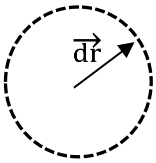
$$S = \omega_S\, (t - \vec{v} \cdot \vec{r}\, /\mathbf{c}^2) = (m/\hbar)\, (\mathbf{c}^2\, t - \vec{v} \cdot \vec{r}\,)$$

We can immediately see that the velocity of the particle is expressed by the relation :

$$\vec{v} = -\, (\hbar/m)\; \overrightarrow{\mathrm{grad}}\; S \qquad (3.12)$$

The phase wave in equation (3.8') is written as :

$$\boldsymbol{P} = \cos\,(S - 2\,\omega_S\, R_g\,/\mathbf{c})$$



Consider the sphere of radius dr centred on the position of the particle at a given instant. On this sphere, the value of the phase wave is $\boldsymbol{P} + d\boldsymbol{P}$, with :

$$d\boldsymbol{P} = -\sin\,(S - 2\,\omega_S\, R_g\,/\mathbf{c})\; dS$$

or: $$d\boldsymbol{P} = -\sin\,(S - 2\,\omega_S\, R_g\,/\mathbf{c})\;(\,\overrightarrow{grad}\; S\,.\,\overrightarrow{dr}\,)$$

(3.12) → $$d\boldsymbol{P} = (m/\hbar)\,\sin\,(S - 2\,\omega_S\, R_g\,/\mathbf{c})\;(\,\vec{v}\,.\,\overrightarrow{dr}\,)$$

On the trajectory of the particle, we have : $\overrightarrow{dr} = \vec{v}\; dt$. This implies that $d\boldsymbol{P}$ reaches its extremum.

We can therefore consider that the particle moves, each time the field is adjusted, in the direction of the extremum of the phase wave (on the Schwarzschild limit).

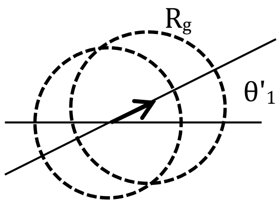
In reference frame Σ, at time t_0, the particle has abscissa x_0. The points of the associated Schwarzschild sphere have abscissa :

$$x = x_0 + R_g \cos\theta'$$

Let's take equation (3.8) again. On the Schwarzschild sphere at time $t_1 = t_0 + R_g\,/\mathbf{c}$, with $r' = R_g$, it becomes :

$$U\,(x_0\,,\theta',t_1) = -\,2\,A(R_S)\,\sin(3\,\omega_S\,R_g\,/\mathbf{c})\,\cos(\omega_S\,(t_0 - R_g\,/\mathbf{c} - v(x_0 + R_g\cos\theta')\,/\mathbf{c}^2\,)) \qquad (3.13)$$

We can see immediately that the derivative $\partial U/\partial\theta'$ has $\sin\theta'$ as a factor and cancels out for $\theta' = 0$. U therefore has an extremum for $\theta' = 0$. In the absence of any disturbance to the wave propagation, this corresponds to a rectilinear displacement.



In the case of interaction, the derivative may cancel for an angle $\theta'_1 \neq 0$ and another angular coordinate φ_1 must be introduced. The coordinates of the particle at time t_1 will be :

$$(x_1 = x_0 + v\,R_g\cos\theta'_1\,/\mathbf{c},\; \theta'_1\,,\; \varphi_1\,)$$

The trajectory is therefore built up in successive sections. In the interpretation proposed above, these are straight line segments, associated with field adjustments as in the analysis developed in chapter 2.

Note: it is the amplitude distribution of the pilot wave at the Schwarzschild limit that controls the motion of the particle. Any perturbation of this distribution can result in a deviation of the trajectory. Such a deviation can be the result of interference phenomena, as explained in the next paragraph.

3.3.2. Interference

The association of the pilot wave with the particle makes it possible to explain interference phenomena as soon as it is accepted that this wave can be diffracted.

In the experiments that have been carried out, interference is obtained by using parameters (slit width and spacing) based on the wavelength of the phase wave. Note that the group wave generally has a much shorter wavelength (in the ratio $\mathbf{v/c}$).

If a screen with two slits is placed in the path of the particles, each particle passes through only one of the slits, but the diffraction of the phase wave causes interference. The result is a deviation in the trajectory of the particle each time the field is refreshed in the direction of the local extremum of the phase wave, as explained in the previous paragraph.

As in Bohmian mechanics, the trajectories depend on the particle's initial conditions. The succession of particles gradually forms the interference pattern.

3.3.3. Differential propagation equation

Note on the Schrödinger equation :

In quantum physics, the Schrödinger equation gives the evolution of the wave function $\psi(\vec{r},t)$ associated with a particle. For a free, non-relativistic particle, it is written :

$$i\,\hbar\,\partial\psi/\partial t = -\,(\hbar^2/2\,m_0)\,\Delta\psi \qquad (3.14)$$

This equation is often presented as admitting a solution of the "De Broglie wave" type:

$$\psi(\vec{r},t) = \psi_0 \exp((-\,i/\hbar)(E\,t - \vec{p}\,\vec{r}\,))$$

This is a misnomer because, although the wave in question does have a wavelength equal to $\mathbf{h}/m_0$ v, its frequency and speed are completely different from De Broglie's hypothesis. This is because the energy taken into account is the kinetic energy and not the total energy of the particle ($E = m\mathbf{c}^2$).

Can the propagation of a wave that fully satisfies the de Broglie hypothesis be described by the Schrödinger equation? On the basis of work carried out by Asim Barut, Laurent Bindel has proposed an affirmative answer to this question: a possible solution to the Schrödinger equation is a de Broglie plane wave modulated in amplitude by a travelling wave solution to a Helmholtz equation.[25]

Inspired by this approach, we will show that the pilot wave is the solution of a differential equation close to the Schrödinger equation.

[25] L. Bindel, *Non-Relativistic Quantum Mechanics of an Individual Particle*, Annales de la Fondation Louis de Broglie, Volume 37, 2012.

Solution using a modified Schrödinger equation :

Let us show that the wave function (3.7) (3.8) is a solution of an equation obtained :

- by writing Schrödinger's equation with the mass m, representative of the total energy, and not the rest mass m_0 ;
- by adding a ψ term to the second member:

$$i \hbar \, \partial\psi / \partial t = -(\hbar^2 / 2m)\Delta\psi + \alpha \psi \qquad (3.15)$$

The propagation equation we are going to establish will be valid whatever the speed of the particle, relativistic or non-relativistic.

Put into complex form, the wave function (3.7) is the product of two functions: $\psi = \boldsymbol{G}\,\boldsymbol{P}$,

with : $\boldsymbol{G} = -2\,A(r')\sin(\omega_S\,(r'/\mathbf{c} + 2R_g/\mathbf{c}))$ (group wave)

$\boldsymbol{P} = \exp(-i(\omega_S\,(t - \vec{v}\,\vec{r}/\mathbf{c}^2 - 2R_g/\mathbf{c}))$ (phase wave)

$= \exp((-i/\hbar)(E\,t - \vec{p}\,\vec{r} - 2\,E\,R_g/\mathbf{c}))$

We have : $\partial\psi/\partial t = (\partial\boldsymbol{G}/\partial t)\,\boldsymbol{P} - i\,\omega_S\,\boldsymbol{G}\,\boldsymbol{P}$

Since the partial derivatives of $\boldsymbol{P}$ with respect to the y and z coordinates are zero, the Laplacian of ψ reduces to :

$$\Delta\psi = \Delta\boldsymbol{G}\;\boldsymbol{P} + 2\,(\partial\boldsymbol{G}/\partial x)\,(\partial\boldsymbol{P}/\partial x) + \boldsymbol{G}\,(\partial^2\boldsymbol{P}/\partial x^2)$$

with : $\partial\boldsymbol{P}/\partial x = i\,(\omega_S\,v/\mathbf{c}^2)\,\boldsymbol{P}$ et $\partial^2\boldsymbol{P}/\partial x^2 = -(\omega_S\,v/\mathbf{c}^2)^2\,\boldsymbol{P}$

or : $\Delta\psi = (\Delta\boldsymbol{G} + 2i\,(\omega_S\,v/\mathbf{c}^2)\,(\partial\boldsymbol{G}/\partial x) - (\omega_S\,v/\mathbf{c}^2)^2\,G)\,\boldsymbol{P}$

By separating the real and imaginary parts, equation (3.12) can be reduced to the system of equations:

$$\Delta\boldsymbol{G} + (2\,(m/\hbar)(\omega_S - \alpha/\hbar) - (\omega_S\,v/\mathbf{c}^2)^2)\,\boldsymbol{G} = 0$$

$$(\partial\boldsymbol{G}/\partial t) + (\hbar/m)\,(\omega_S\,v/\mathbf{c}^2)\,(\partial\boldsymbol{G}/\partial x) = 0$$

Taking into account the relationship $\omega_S = m\mathbf{c}^2/\hbar$, these equations become :

$$\Delta\boldsymbol{G} + (2\,((\omega_S/\mathbf{c})^2 - (\alpha/\hbar)\,\omega_S/\mathbf{c}^2) - (\omega_S/\mathbf{c})^2(v^2/\mathbf{c}^2))\,\boldsymbol{G} = 0 \qquad (3.16)$$

$$(\partial\boldsymbol{G}/\partial t) + v\,(\partial\boldsymbol{G}/\partial x) = 0 \qquad (3.17)$$

Equation (3.17) is satisfied by $\boldsymbol{G}$ because this function has the variable :

$r' = ((x - v\,t)^2 + y^2 + z^2)^{1/2}$ So $\boldsymbol{G}$ is a function of $(x - v\,t)$.

Let us find α such that the coefficient of $\boldsymbol{G}$ in equation (3.16) is equal to $(\omega_S/\mathbf{c})^2$:

$$(\omega_S/\mathbf{c})^2(1 - v^2/\mathbf{c}^2)) = 2\,(\alpha/\hbar)\,\omega_S/\mathbf{c}^2$$

$$\alpha = \hbar\,\omega_S\,(1 - v^2/\mathbf{c}^2)/2 = m(\mathbf{c}^2 - v^2)/2$$

Equation (3.16) is finally written :

$$\Delta \boldsymbol{G} + (\omega_S / \mathbf{c})^2 \boldsymbol{G} = 0 \tag{3.18}$$

This is a Helmholtz equation.

With $A(r') = - 2 \text{ G m} / r'$ (see paragraph 3.2.1.), the $\boldsymbol{G}$ function is written as :

$$\boldsymbol{G} = - 4 (\text{G m} / r') \sin ((\omega_S / \mathbf{c})(r' + 2R_g))$$

It has a pulsation equal to $\omega_S / \mathbf{c}$ and verifies equation (3.15): it can be identified with a spherically symmetric solution of this equation.

The function describing the pilot wave in reference frame Σ is therefore a solution of equation :

$$i \hbar \, \partial\psi / \partial t = - (\hbar^2 / 2 \text{ m}) \Delta\psi + \text{m} (\mathbf{c}^2 - v^2) \psi / 2 \tag{3.19}$$

3.3.4. The limit of wave behaviour

We have seen that the frequency linked to the energy of the gravitational energy quanta (ν_S) and the emission frequency ($1/2T$) are distinct.

The ratio between these frequencies is :

$$\nu_S / (1/2T) = (m\mathbf{c}^2 / \mathbf{h}) / (\mathbf{c}^3 / 4 \text{ Gm}) = 4 \text{ m}^2 (G / \mathbf{hc})$$

By introducing the Planck mass $[m_p = (\hbar \mathbf{c} / G)^{1/2}]$, this ratio is written :

$$\nu_S / (1/2T) = (2/\pi) (m/m_p)^2 \tag{3.20}$$

If we refer to the ratio defined by equation (3.20), we can see that the experiments used to verify De Broglie's hypothesis have all involved particles with masses very small compared with the Planck mass, for which the frequency of the gravitational waves is much lower than the refresh rate. **Perhaps the ratio of 1 constitutes a limit for the manifestation of wave-like behaviour in particles?**

At the stage we have reached, we can conclude that it is possible to explain the behaviour of a moving particle by its gravitational properties:

- **modification of its speed in the event of a modification of its energy, in accordance with the fundamental law of dynamics ;**
- **wave behaviour associated with the gravitational waves that accompany it.**

Appendix

Principles used to establish the laws of gravitation for a non-zero mass particle in a weak field[26]

First principle: gravitational field and potential energy

The gravitation created by a gravitational source of mass m can be characterised by a gravitational field that imparts potential energy to the particles present in it.

The variation in the potential energy of a particle, of zero or non-zero mass, is proportional to its total energy E, calculated in the reference frame linked to the source; it is given by the relation :

$$dE_g = 2\ (Gm/\ \mathbf{c}^2 r^2)\ E\ dr$$

Second principle: conservation of energy

The variation in total energy of a particle is equal to the opposite of the variation in potential energy plus the work of external forces, if any:

$$dE = dE_x - dE_g$$

Third principle: the influence of gravitation on a particle's energy and momentum

The rest energy of a non-zero mass particle varies with its distance from the source.

In the absence of external forces, the variation in energy associated with rest energy and the variation in energy associated with momentum are each equal to and opposite half the variation in potential energy.

Fourth principle: equivalence between gravitation and acceleration

The fundamental law of dynamics can be applied to determine the relationship between the change in momentum of the particle and the change in energy associated with that momentum under the effect of the gravitational field.

(Given the third principle, equivalence cannot be considered complete)

[26] cf. note *"Another approach to relativity"*, § 5.1.2.