Another approach to relativity

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1. Introduction and summary

In the theory of special relativity, the principle of relativity (invariance of the laws of physics when changing a Galilean reference frame) and the universality of the speed of light (independence from the reference frame and from the velocity of the source) imply that the clocks of two reference frames in relative motion appear to be out of sync. Apart from this desynchronisation, the equations for changing coordinates (Lorentz formulae) show a dilation of durations and a contraction of distances from one reference frame to the other.

Several physical phenomena, such as the delay of clocks in motion or the increase in the lifetime of atmospheric muons compared with muons at rest, are in agreement with the predictions of the theory and give reality to time dilation, according to the commonly accepted interpretation. It is implied that the systems under consideration are not affected by their rectilinear and uniform motion: in particular, clocks continue to deliver the same unit of time.

From our point of view, this phenomenon of time dilation constitutes a real point of contradiction, since observers of one reference frame are justified in maintaining that time really does pass more slowly in any reference frame in rectilinear and uniform motion compared with their own reference frame.

For its part, the theory of general relativity uses a curvature of space-time, produced by the distribution of energy, to account for gravitation. In this way, it fully explains phenomena such as the advance of Mercury's perihelion or the curvature of light rays close to the sun, which can be observed during eclipses.

The starting point for this paper is the search for an answer to the following question: **without calling into question the principle of relativity, is it possible to construct an alternative, non-contradictory theory that does not involve distortions of space and time?**

After a reminder of the synchronisation of clocks in Galilean reference frames and the establishment of the equations for changing coordinates between reference frames within the framework of special relativity, we examine the concept of event and propose an alternative analysis that is consistent with an absence of dilation of space and time. The change of coordinates is no longer one-to-one.

On the basis of the principle of equivalence between mass and energy and of the fundamental law of (relativistic) dynamics, we propose a second analysis that supports the previous one. It leads us to question the principle of invariance of mass at rest, which is accepted in the theory of special relativity as it is in classical mechanics. We present an alternative to this principle: the energy imparted to a particle (in a given reference frame) to set it in motion is conserved in the reference frame where this particle is at rest. The invariant is no longer the mass at rest, but the total energy of the particle. This hypothesis is consistent with the absence of dilation of space and time.

The phenomena mentioned above then appear to be the consequence, not of time dilation, but of the variation in energy at rest from one frame of reference to another. Since the atoms in the clock that is set in motion have greater energy than those in the clock that remains stationary, these two clocks no longer deliver the same unit of time[1] .

One chapter is devoted to the consideration of luminous phenomena, comparing the new approach we are proposing with that of special relativity.

The hypothesis of non-invariance of rest mass naturally leads us to consider that gravitation can have an influence on it: the gravitational shift of clocks is interpreted as a consequence of the variation in the rest energy of atoms as a function of their distance from the gravitational source. This implies that gravitation and acceleration cannot be considered to be completely equivalent.

We show that this hypothesis makes it possible, in the case of weak gravitational fields, to formulate simple laws while remaining within the framework of dynamics without space-time deformation:

- the variation in the potential energy of a particle, with zero or non-zero mass, is proportional to its total energy;
- the variation in total energy of a particle is equal to the opposite of the variation in potential energy plus the work of external forces, if any;
- the rest energy of a particle of non-zero mass varies with its distance from the source. In the absence of external forces, the variation in energy associated with rest energy and the variation in energy associated with momentum are each equal to half the opposite of the variation in potential energy;
- the fundamental law of dynamics can be applied to determine the relationship between the variation in the momentum of a particle of non-zero mass and the corresponding variation in energy under the effect of the gravitational field;
- particles of zero mass are slowed down as they pass through the gravitational field.

[1] at least for an atomic clock

With these laws, the physical phenomena that have been experimentally verified are correctly explained, including the Shapiro effect. There is a difference with the theory of general relativity in the evaluation of the gravitational spectral shift, but the result of the Pound and Rebka experiment is still explained.

In conclusion, the note explains the similarities and differences with the theory of general relativity by referring to the Schwarzschild metric.

One conclusion, valid for both special and general relativity, could be as follows: the postulate of invariance of mass at rest obliges us, in these theories, to deform space and time in order to simulate the invariance of total energy taken into account in the proposed new approach.

A gravitational field model justifying the laws proposed for weak-field gravitation is presented in a separate note entitled "Gravitational Field, Fundamental Principle of Dynamics and Quantum Mechanics". This model also makes it possible to take into account a problem that is not limited to two bodies and to envisage an extension of the laws outside the weak field.

Of course, there is still a fundamental question that may call into question our current representations of matter and energy: since we accept that particles of the same type can have different rest energies, what does this difference in energy physically correspond to?

2. Galilean reference frames and changing reference frames

2.1. A reminder of the properties of Galilean reference frames

An observer is aware of the space that surrounds him and of the time that passes. He can situate an event_ i.e. a physical phenomenon[2] occurring at a point in space_ in his own frame of reference:

- firstly, spatially in a reference frame giving space coordinates using graduations based on a length standard;
- secondly, in time, thanks to clocks[3] placed at every point of the reference frame, delivering a unit of time identical to that of a standard clock;

Galilean reference frames are those in which space appears homogeneous and isotropic and time uniform with respect to physical phenomena, which means that these phenomena are unchanged by the operations of translation in space, rotation in space and translation in time. All observers in such a frame of reference are equivalent.

The very definition of Galilean reference frames dictates that two Galilean reference frames in relative motion are necessarily in rectilinear and uniform spatial translation motion relative to each other.

If only one reference frame is used, the length standard and the time standard can be chosen arbitrarily. On the other hand, to compare measurements made in two reference frames in motion relative to each other, it is necessary to refer to standards common to both reference frames.

We therefore postulate the existence of such standards that deliver the same units of spatial distance and time in all reference frames. The units of space and time are universal.

On the other hand, we retain the principle of relativity: the laws of physics are invariant to any change of Galilean reference frame.

2.2. Clock synchronisation

The synchronisation of clocks makes it possible to define the notion of simultaneity of two events occurring at two different points: in a Galilean reference frame, two events are said to be simultaneous if they occur at the same instant given by the clocks placed at these points.

It is an essential prerequisite for describing the motion of a moving object as it moves from one point to another, and in particular for determining its velocity.

The fact that, in a Galilean frame of reference, two identical experiments, shifted by translation and rotation in space, take place with the same durations, theoretically allows clocks to be synchronised from any process that can be reproduced identically at different points and in different directions.

[2] A physical phenomenon can be simply the positioning of a particle at a given point in time.
[3] A clock is a physical device that samples time using a succession of identical cycles.

Because of the uniqueness of time in a given reference frame, any truly invariant process necessarily leads to the same synchronisation, which is therefore unique. Synchronisation can, for example, be based on the rectilinear motion of a moving body whose velocity is assumed to be constant whatever its position and direction of motion. It is not essential to exchange light rays.

Let's imagine that we have synchronised clocks separately in two Galilean reference frames in relative motion. If, at a given instant t in one frame of reference, all the clocks in the other frame of reference mark the same time t' (which can therefore be equal to t by an ensemble shift), time is considered to be absolute.

The equations linking the space and time coordinates of the same event in two Galilean reference frames are then the (so-called Galilean) equations of Newtonian Mechanics. The law of addition of velocities associated with these equations imposes no limit on the speed of a moving body.

Conversely, as we will show below, if the times appear to be out of sync from one frame of reference to another, a limiting speed has to be considered.

2.3. Change of reference frame in the theory of special relativity

We will now look at how the equations for changing reference frames are established in the theory of special relativity.

2.3.1. Taking desynchronisation into account

Consider two Galilean reference frames Σ (x, y, z, t) and Σ' (x', y', z', t') in motion relative to each other parallel to the axis (x). Let t = t'= 0 when the origins of the reference frames coincide; let u be the velocity of Σ' relative to Σ.

The theory of special relativity assumes that reference frames are indistinguishable from one another, including in the writing of coordinate equations. There is no such thing as a privileged frame of reference. Note that this assumption goes beyond the strict principle of relativity, which simply implies that an experiment is described by the same physical laws in each frame of reference.

The same event is perceived at the point (x, y, z, t) of Σ and at the point (x', y, z, t') of Σ'.[4] Note that, since the reference frames are Galilean, the need for invariance by translation in space and time means that the equations for changing coordinates must be linear, of the form below (k being a coefficient of inverse dimension of a speed):

$$x' = a\,(x - u\,t) \qquad (2.1)$$
$$t' = b\,(t - k\,x)$$

We assume that the clocks appear to be out of sync from one frame of reference to another. This implies that the coefficient k is not zero.

[4] The notion of perception of an event is explained in paragraph 2.4.1.

Desynchronisation means that at time 0 in one frame of reference, the clocks in the other frame of reference mark a non-zero time (apart from that at the origin), varying linearly with their abscissa.

The inversion of equations (2.1) leads to :

$$x = (1/(1 - k\,u))(x'/a + u\,t'/b) \qquad (2.1')$$
$$t = (1/(1 - k\,u))(t'/b + k\,x'/a)$$

Independently of any experiment, it is logical to consider that, since the velocity of the frame of reference Σ' relative to Σ is equal to u, the velocity of Σ relative to Σ' is equal to - u.

It follows that: $b = a$

Equations (2.1) become:

$$x' = a\,(x - u\,t) \qquad (2.2)$$
$$t' = a\,(t - k\,x)$$

2.3.2. Existence of a limiting speed

Using equations (2.2), we can write:

$$dx'/dt' = (dx - u\,dt) / (dt - k\,dx) = (dx/dt - u) / (1 - k\,dx/dt)$$

Whis: $v = dx/dt$ and $v' = dx'/dt'$:

$$v' = (v - u) / (1 - k\,v) \qquad (2.3)$$

If we want to avoid a discontinuity for v' (from $-\infty$ to $+\infty$) and given that the value 0 necessarily belongs to the permitted interval for v, we must have :

$v < 1/k$ [assuming $k > 0$].

By reasoning about the inverse of relation (2.3), we also obtain:

$$v' > -1/k$$

The isotropy of Galilean reference frames means that, if the speed has a limit in one direction, it must have the same limit in the opposite direction. It is not necessary to assume that the limit must be the same in both reference frames. The velocities v and v' must therefore satisfy a double inequality:

$$-w < v < w \qquad -w' < v' < w' \qquad (2.4)$$

w and w' being the limiting speeds, which must be less than $1/k$ [assuming $k > 0$].

Finally, because of the equivalence of the reference systems,

the condition: $v \to w$ leads to: $v' \to w'$

the condition: $v \to - w$ leads to: $v' \to - w'$

Applied to equation (2.3), these conditions imply:

$$(w - u) / (1 - k w) = (w + u) / (1 + k w)$$

from which we derive: $k = u/w^2$

and: $w' = w$

The limiting speed is the same in both reference frames.

$$w < 1/k \quad \to \quad u < w$$

The relative speed of the reference frames is necessarily less than the limiting speed.

The law of composition of velocities is finally written:

$$v' = (v - u) / (1 - u v/w^2) \qquad (2.5)$$

2.3.3. Lorentz formulae

The equations for moving from coordinates in Σ to coordinates in Σ' are written as:

$$x' = a (x - u t) \qquad (2.6)$$
$$t' = a (t - u x/w^2)$$

The inverse equations are:

$$x = a' (x' + u t') \qquad (2.6')$$
$$t = a' (t' + u x'/w^2)$$

where: $a a' = \Upsilon^2 = 1/(1- u^2 /w^2)$

If we take: $w = \mathbf{c}$, the speed of light, and $a = a' = \Upsilon$ (so as not to differentiate between reference frames), we obtain Lorentz's formulae:

$$x' = \Upsilon (x - u t) \qquad x = \Upsilon (x' + u t') \qquad (2.7)$$
$$t' = \Upsilon (t - u x / \mathbf{c}^2) \qquad t = \Upsilon (t' + u x'/ \mathbf{c}^2)$$

We can see that it is not essential to start from the postulate of the invariance of the speed of light in order to establish this result. It is the existence of a limiting speed (underpinned by the desynchronisation of the clocks) which, together with the hypothesis of non-differentiation of reference frames, constitutes the key element.

2.3.4. Expansion or contraction of space and time

With equations (2.7), apart from time desynchronisation, we observe:

- a dilation of the duration between two events, measured using two different clocks (improper time) compared with the duration measured by a single clock (proper time), i.e. in the frame of reference where these events have the same spatial position:

$$\Delta t' = \Upsilon\, \Delta t \text{ if } \Delta x = 0 \quad \text{and} \quad \Delta t = \Upsilon\, \Delta t' \text{ if } \Delta x' = 0 \;;$$

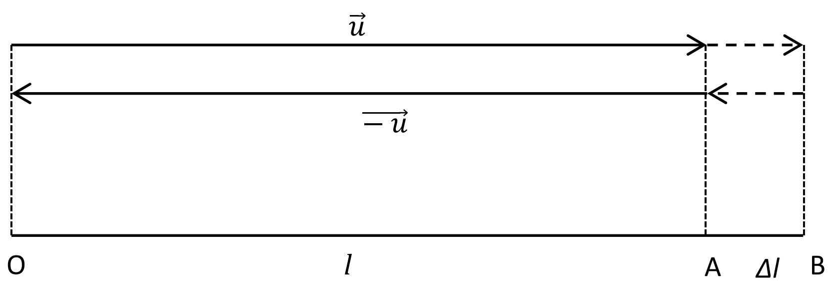
- a contraction of the lengths measured from the moving frame of reference relative to the lengths in the fixed frame of reference:

$$\Delta x' = \Delta x / \Upsilon \text{ if } \Delta t' = 0 \quad \text{and} \quad \Delta x = \Delta x' / \Upsilon \text{ if } \Delta t = 0.$$

The reality of time dilation

Does the dilation of time and its corollary, the contraction of length, have a physical reality or is it an apparent effect linked to the desynchronisation of clocks?

Consider the so-called "clock paradox" experiment [5] :



Consider two Galilean reference frames: Σ and Σ' moving at velocity u relative to Σ.

In the reference frame Σ, a clock is set in motion so that it reaches speed u and is in front of O and O' at the moment of their conjunction; it moves from O to A at speed u, is slowed down to zero speed at B , then accelerated in the opposite direction to the speed - u which it maintains from A to O. The moving clock is synchronised with the clocks at O and O', so that all three clocks show the same time (= 0) at O,O'.

If we consider that, once set in rectilinear motion at constant velocity, the mobile clock continues to deliver the same unit of time as the standard clock, it is perfectly comparable, during its outward journey OA, to the clock at the origin O' of the reference frame Σ'.

When the moving clock reaches point A, the times given by the clocks are (according to equations (2.7)):

clock A: t_1 (= l/u) moving clock: $t_1' = t_1 / \Upsilon$

[5] or "twin paradox".

The moving clock therefore lags behind the fixed clock opposite it by the ratio $1/\Upsilon$.

When she returns to A after turning around, the times are :

clock A: $t_2 = t_1 + \Delta t$ moving clock: $t'_2 = t_1 / \Upsilon + \Delta t'$

(Δt and $\Delta t'$ are the durations induced by the half-turn of the mobile clock)

By symmetry, the return journey of the moving clock has the same proper duration as the outward journey; moreover, as the clocks at O and A are synchronised in Σ, the durations measured in this reference frame are also identical [6] .

The arrival times are: clock in O: $t_3 = 2\,t_1 + \Delta t$ moving clock: $t'_3 = 2\,t_1 / \Upsilon + \Delta t'$

Δt and $\Delta t'$ are fixed durations, independent of t_1 ; they become negligible compared with t_1 as soon as the latter is sufficiently large. It can be said that, on its return journey, the mobile clock should show an offset in relation to the clock remaining in A, in a ratio $1/\Upsilon$ identical to that of the outward journey.

Experiments at [7] confirm the physical reality of the total time lag and therefore of the time lag observed on the outward journey. As a result, observers in Σ' not only have the impression that durations in Σ' are shorter than those in their own frame of reference; for them, time really does pass more slowly in Σ' than in Σ. But the opposite assertion can just as easily be made by observers in Σ' (all they have to do is perform the exactly symmetrical experiment from Σ').

In the end, this is not a paradox but an effective contradiction that is accepted by the theory of special relativity. The time dilation hypothesis makes it possible to construct a theory that is mathematically coherent and consistent with observations. But this theory is physically contradictory.[8]

2.4. Alternative analysis to that of special relativity

2.4.1. Principles

How can we avoid the contradiction that has just been highlighted?

A simple assumption is that time flows universally (which means that there is no dilation of durations) but that the synchronisation of clocks does not allow it to be expressed as absolute time, because of the existence of a limiting speed. On the other hand, space is considered to be absolute (no contraction of lengths between reference frames).[9]

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[6] The mobile clock moves identically in both directions and therefore complies with the conditions for a synchronisation experiment (see 2.2.).

[7] These are experiments carried out with atomic clocks (the first one being by Hafele and Keating in 1971).

[8] The objection was raised in 1922 by Painlevé against Einstein (see Vincent Borella, "A propos du paradoxe de Langevin", *Philosophia Scientiæ*, vol. 1, no. 1 (1996), pp. 63-82). It has been repeated many times since. Several solutions have been proposed to rule it out, but no general agreement has been reached.

[9] Let us observe that the conservation of distances and durations when changing reference frames could constitute a necessary condition for the validity of the principle of relativity.

Galilean reference frames in relative motion are linked by the definition of a common instant of time origin when the origins of the spatial reference frames coincide.

Assuming the universality of time, every event is characterized by the duration T, which separates if from the common instant of origin. We will say that an event is "produced" in a reference frame if the time displayed by the clock at the point to which it is attached is identical to that duration.

Consider an event “produced” at point (x , T) in frame of reference Σ. In a reference frame Σ' moving relative to Σ, the event is perceived at (x', t'). Because of the desynchronisation of the clocks, t' is different from T (unless $x' = 0$); but, for the observers of Σ, the time elapsed at the clock of abscissa x' is indeed equal to T (we will see in the next paragraph what are the equations for changing coordinates that allow us to conserve times and distances).

Now we must understand that, if we start from an event “produced” in Σ' at the point with coordinates (x', t'), this event will not be perceived in Σ at (x, T) but at a point with different coordinates (x_1 , t_1). This is not surprising, since the event in Σ' occurs after a time T'= t' ($\neq T$).

The change of coordinates is no longer one-to-one, as is the case in the theory of special relativity. This is because an event, considered as "produced" in the frame of reference where the clock displays the duration that has actually elapsed, is "perceived" at a different time in another moving frame of reference, and we introduce no correction for time dilation.

2.4.2. New coordinate change equations

Let's start with an event “produced” at point (x, t) in Σ. Let's take equations (2.6):

$$x' = a\,(x - u\,t)$$
$$t' = a\,(t - u\,x / w^2)$$

It is easy to check that the absence of dilation of distances and durations is simply expressed by a = Υ^2. Then we have:

$$\Delta x' = \Delta x \ \text{ if } \ \Delta t' = 0 \qquad \text{and} \qquad \Delta t' = \Delta t \ \text{ if } \ \Delta x' = 0$$

Measured in the Σ' reference frame, distances and durations are indeed equal to what they are in Σ.

The coefficient Υ^2 reflects the fact that, at a given instant in Σ, this reference frame corresponds to a distorted image of Σ' (the clocks appear to be out of sync), which is a kinematic effect. Whereas, in the theory of special relativity, the Υ coefficient reflects desynchronisation coupled with a real deformation of space and time ($\Upsilon = \Upsilon^2 / \Upsilon$).

The contradiction highlighted in paragraph 2.4.2 (clock paradox) disappears.

Important consequence: **the delay of a moving clock can no longer be attributed to time dilation.** [10]

[10] See section 3.3.1.

Finally, the coordinate change equations can be written as :

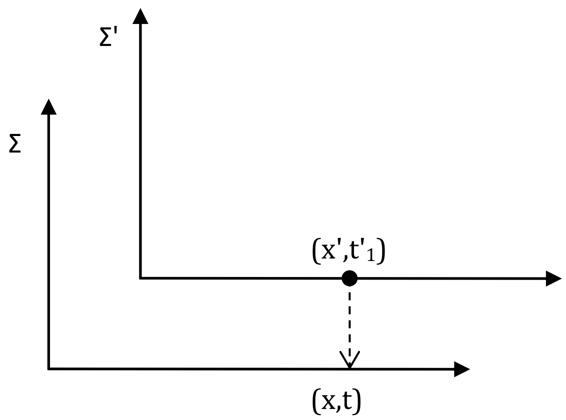
- for an event "produced" in reference frame Σ :

$$x' = \Upsilon^2 (x - u\,t) \qquad x = x' + u\,t' \qquad (2.8)$$
$$t' = \Upsilon^2 (t - u\,x / \mathbf{c}^2) \qquad t = t' + u\,x' / \mathbf{c}^2$$

- for an event "produced" in reference frame Σ' :

$$x = \Upsilon^2 (x' + u\,t') \qquad x' = x - u\,t \qquad (2.8a)$$
$$t = \Upsilon^2 (t' + u\,x' / \mathbf{c}^2) \qquad t' = t - u\,x / \mathbf{c}^2$$

How can the equations above be applied to describe the conjunction of points with coordinates x in Σ and x' in Σ'?

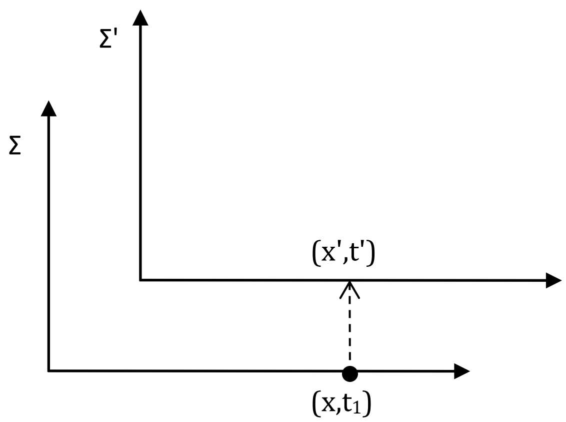


For each observer, the conjunction of the coordinates x and x' is perceived as an experience taking place in his own frame of reference.

For the observer of Σ, this is the movement at velocity u of a moving body (x') passing through x at time t. Applying equations (2.8) :

$$x = x' + u\,t'_1 \qquad (2.9)$$
$$t = t'_1 + u\,x'/\mathbf{c}^2$$

For the observer of Σ', this is the displacement at velocity - u of a moving body (x) passing through x' at time t'.
Applying equations (2.8a) :



$$x' = x - u\,t_1 \qquad (2.10)$$
$$t' = t_1 - u\,x/\mathbf{c}^2$$

It comes immediately:

$$t_1 = t'_1 = (x - x')/u = t'' \qquad (2.11)$$

t_1 and t'_1 therefore represent the time t'' given by the Galilean transformation.

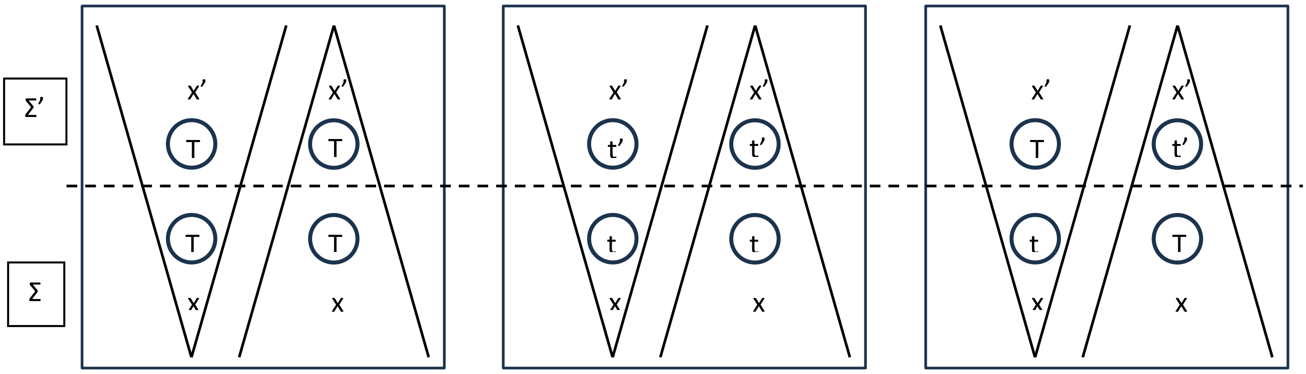
We have: $t = t'' + u\,x'/\mathbf{c}^2$ and $t' = t'' - u\,x/\mathbf{c}^2$

It is important to note that we are dealing with two different experiences.

Finally, what we have just said can be translated into the following statement:

The conjunction of the coordinates x in reference frame Σ and x' in reference frame Σ' constitutes a double event, produced at different times t and t'. On the other hand, the two events are perceived at an identical time in the two reference frames, equal to the duration given by the Galilean transformation.

The diagrams below illustrate the perception of the conjunction of coordinates x and x' according to whether we take Newtonian mechanics, special relativity or the proposed new approach. Imagine that contact between x and x' triggers the taking of two photos, one from frame of reference Σ (at x), the other from frame of reference Σ' (at x').



Mécanique newtonienne | Relativité restreinte | Nouvelle approche

$T = (x - x')/u$

$t = (x - x'/\gamma)/u$

$t' = (x/\gamma - x')/u$

$t = (x - x'/\gamma^2)/u$

$t' = (x/\gamma^2 - x')/u$

<u>Let's now consider the case of an event "produced" in one frame of reference Σ and "perceived" in two other frames of reference Σ' and Σ''</u>.

Let u' be the velocity of Σ' relative to Σ, u'' the velocity of Σ'' relative to Σ.

From equations (2.8) we obtain the equations for changing coordinates from Σ' to Σ'' :

$$x'' = a'' (x' - U t') \qquad \text{where: } a'' = (1 - u' u''/w^2)/(1 - u''^2/w^2)$$

$$t'' = a'' (t' - U x'/w^2) \qquad U = (u'' - u')/ (1 - u' u''/w^2)$$

Finally, what can we say about an event "perceived" in Σ' and Σ'' without knowing in advance the frame of reference in which it is "produced"?

Knowing the coordinates (x', t') and (x'', t'') of the event, we can use the equations above to determine the values of a'' and U and then those of u' and u'', thus defining the event's reference frame.

<u>Conclusion:</u>

In Newtonian mechanics, there is no distinction between "produced" and "perceived" events, because time is absolute.

In the theory of special relativity, no such distinction is made either, by imposing a one-to-one change of coordinates. This amounts to assuming that the conjunction of two points with coordinates x in Σ and x' in Σ' constitutes the same event "produced" simultaneously in both reference frames.

This is what leads to the contradiction: if time is not absolute, an event cannot be considered as "produced" in all frames of reference, with the exception of the event used to define the common instant of origin.

3. Relativistic approach based on the equivalence between mass and energy

This chapter sets out an alternative approach to the problem, based on energy considerations, which provides justification for the alternative analysis presented in sub-chapter 2.4.

3.1. Relationship between energies calculated in two Galilean reference frames

3.1.1. Particle physics has validated the principle of equivalence between mass and energy.

We will show that we can deduce the existence of a limiting speed from this principle.

We postulate that, in any Galilean reference frame Σ (x, y, z, t)**, there is the following relationship between the mass**[11] m **of a particle and its total energy** E **:**

$$E = m\,\mathbf{c}^2 \qquad (3.1)$$

m depends on the velocity of the particle. This is the inertial mass that comes into play in the definition of the particle's momentum.

The factor **c** is a coefficient that is independent of m and has the dimension of a speed. For the moment, **c** does not necessarily refer to the speed of light because the reasoning that follows does not refer to light rays.

3.1.2. We restrict ourselves to a one-dimensional problem by considering a particle moving parallel to the axis (x). This particle is subjected to the action of a force F also parallel to the axis (x). Let v be the velocity of the particle.

The fundamental equation of dynamics is written:

$$d(mv)/dt = F \qquad (3.2)$$

The change in energy is equal to the work of the force:

$$dE = F\,dx, \qquad \text{i.e.} \quad dE/dt = F\,dx/dt = F\,v$$

or:

$$\mathbf{c}^2\,dm/dt = F\,v \qquad (3.3)$$

From equations (3.2) and (3.3) we derive:

$$\mathbf{c}^2\,dm/dt = v\,d(mv)/dt$$

$$\mathbf{c}^2\,dm/dt = mv\,dv/dt + v^2\,dm/dt$$

$$(dm/dt)/m = v\,(dv/dt)/(\mathbf{c}^2 - v^2) = -\,(d(\mathbf{c}^2 - v^2)/dt)/(\mathbf{c}^2 - v^2)/2$$

[11] The term "mass" is not reserved for rest mass.

Integrating gives: $\ln(m) = \ln((\mathbf{c}^2 - v^2)^{-1/2}) + \text{constant}$

And, finally, let m_0 denote the rest mass of the particle in Σ:

$$m = m_0 / (1 - v^2 / \mathbf{c}^2)^{1/2} \qquad (3.4)$$

Therefore: $$E = m_0 \mathbf{c}^2 / (1 - v^2 / \mathbf{c}^2)^{1/2} \qquad (3.5)$$

This expression for E is the one verified by experiments.

c appears to be the limit of the speed of a non-zero mass particle.

On the other hand, if we denote the momentum by p (= mv), we have the relation:

$$E^2 - p^2 \mathbf{c}^2 = m_0{}^2 \mathbf{c}^4 \qquad (3.6)$$

3.1.3. Let us now consider a second Galilean reference frame Σ' (x', y', z', t') in motion relative to the first, parallel to the axis (x), at velocity u. The relationship between mass and energy is written:

$$E' = m' \mathbf{c}^2 \qquad (E'_0 = m'_0 \mathbf{c}^2 \text{ for the particle at rest in } \Sigma')$$ [12]

The expressions for mass and energy in reference frame Σ' are: [13]

$$m' = m'_0 / (1 - v'^2 / \mathbf{c}^2)^{1/2} \qquad (3.4')$$

$$E' = m'_0 \mathbf{c}^2 / (1 - v'^2 / \mathbf{c}^2)^{1/2} \qquad (3.5')$$

Whis: $\underline{u} = u / \mathbf{c}$ $\quad \underline{v} = v / c$ $\quad \underline{v}' = v' / c$

The law of velocities composition (2.5) is written as:

$$\underline{v'} = (\underline{v} - \underline{u})/(1 - \underline{u}\,\underline{v})$$

This law is well verified by experiments involving interactions between particles.

We therefore have: $(1 - \underline{v}'^2) = ((1 - \underline{u}\,\underline{v})^2 - (\underline{v} - \underline{u})^2)/(1 - \underline{u}\,\underline{v})^2$

or : $$(1 - \underline{v}'^2) = (1 - \underline{u}^2)(1 - \underline{v}^2)/(1 - \underline{u}\,\underline{v})^2 \qquad (3.7)$$

3.1.4. The values of the energy of the particle expressed in the two reference frames are therefore linked by the relation below:

$$E' = (m'_0 / m_0)\ E\ (1 - \underline{u}\,\underline{v})/(1 - \underline{u}^2)^{1/2} \qquad (3.8)$$

[12] The limiting speed **c** is assumed to be identical in all reference frames, so as not to impose any differentiation between them in terms of the laws of physics. On the other hand, the masses at rest do not have to be identical, as we will see in sub-chapter 3.2.

[13] In application of the principle of relativity, the fundamental law of dynamics is expressed in the same way in Σ' as in Σ.

The inverse relationship is written:

$$E = (m_0 / m'_0)\ E'\ (1 + \underline{u}\,\underline{v}')/(1 - \underline{u}^2)^{1/2} \qquad (3.8')$$

3.2. Choosing an energy invariant

Relations (3.8) and (3.8') show the ratio m'_0 / m_0 . The value to be given to this ratio depends on the choice of a relationship between the particle's rest energies in the two reference frames.

There are two reasonable hypotheses:

- the invariance of rest mass (cf. 3.2.1.);
- the conservation of total energy by change of reference frame (cf. 3.2.2.).

3.2.1. Invariance of the rest mass of any particle

The first hypothesis postulates that the rest mass of a particle is a characteristic intrinsic to the particle, and therefore invariant to any change of reference frame. This postulate, which is identical to that of classical mechanics, is also the postulate of special relativity. Energy at rest has the same value in all reference frames.

With $m'_0 / m_0 = 1$, equations (3.8) and (3.8') become :

$$E' = E\ (1 - \underline{u}\,\underline{v})/(1 - \underline{u}^2)^{1/2} \qquad (3.9)$$

$$E = E'\ (1 + \underline{u}\,\underline{v}')/(1 - \underline{u}^2)^{1/2} \qquad (3.9')$$

Given $p = m\,\mathbf{c}\,\underline{v}$ the momentum, and assuming $\Upsilon = 1/(1 - u^2/\mathbf{c}^2)^{1/2}$, we get:

$$E' = \Upsilon\ (E - \underline{u}\ p\ \mathbf{c}) \qquad (3.10)$$

We find the usual relativistic relationship.

Let us show that the rest mass invariance assumption is compatible with Lorentz's formulae. Let's start with the equation of dynamics in Σ' [14]:

$$d(m'v') = F\,dt' \text{ where } m' = \Gamma'\,m_0 \text{ and } \Gamma' = 1/(1 - \underline{v}'^2)^{1/2}$$

or :

$$d(\Gamma' v') = (F/m_0)\,dt'$$

The relation (3.7) and the law of velocities composition (2.5) allow us to write Γ' as :

[14] we admit that, when $\vec{F}$ is collinear with $\vec{v}$, its value is preserved by change of frame of reference.

$$\Gamma' = \Upsilon\, \Gamma\, (1 - \underline{v}\, \underline{u}) \qquad \text{(where} \quad \Gamma = 1/(1 - \underline{v}^2)^{1/2}\text{)}$$

$$\Gamma' v' = \Upsilon\, \Gamma\, (v - u)$$

$$d(\Gamma' v') = \Upsilon\, (d(\Gamma\, v) - u\, d\Gamma) \qquad (3.11)$$

Let us show that: $u\, d\Gamma = d(\Gamma\, v)\, \underline{u}\, \underline{v}$

$$d(\Gamma\, v)\, \underline{u}\, \underline{v} = \Gamma\, \underline{u}\, \underline{v}\, dv + \underline{u}\, \underline{v}\, v\, d\Gamma$$

$d\Gamma = \Gamma^3\, \underline{v}\, dv \rightarrow$ $\quad d(\Gamma\, v)\, \underline{u}\, \underline{v} = u/\Gamma^2\, d\Gamma + u\, \underline{v}^2\, d\Gamma = u\, (1/\Gamma^2 + \underline{v}^2)\, d\Gamma = u\, d\Gamma$

(3.11) $\rightarrow$ $\quad d(\Gamma' v') = \Upsilon\, (d(\Gamma\, v) - \underline{u}\, \underline{v}\, d(\Gamma v)) \qquad (3.11')$

The equation of dynamics in Σ is written:

$$d(\Gamma v) = (F/m_0)\, dt$$

Eliminating F/m_0 $\quad$ (3.11') $\rightarrow$ $\quad dt' = \Upsilon\, (dt - \underline{u}\, \underline{v}\, dt)$

$v = dx/dt \rightarrow$ $\quad dt' = \Upsilon\, (dt - (u/\mathbf{c}^2)\, dx) \qquad (3.12)$

Lorentz's formula for time co-ordinates is clearly applicable.

3.2.2. Conservation of total energy by change of reference frame

To understand this second hypothesis, let's imagine the following experiment:

- two identical particles, with the same rest mass m_0 in Σ, are immobile in this frame of reference; they have a velocity equal to - u in Σ' ;
- an observer of Σ increases the speed of the first particle from 0 to u by supplying it with energy;
- an observer of Σ' reduces the speed of the second particle from - u to 0 by withdrawing energy from it;
- the two particles are now at rest with respect to Σ'; what is the rest mass of each in Σ'?

If the answer is that it remains equal to m_0 , then the mass at rest is invariant.

The second hypothesis postulates that the difference between the rest energies of a particle is equal to the energy supplied to it (or withdrawn from it) in order to move from zero speed in one frame of reference to zero speed in the other frame of reference; the particle's total energy is transferred, as it were, from one frame of reference to the other.

Kinetic energy is therefore no longer considered to be "lost" when moving from one frame of reference to another, as is the case with the assumption of invariance of mass at rest.[15]

More precisely, the rest energy of a particle in reference frame Σ' is equal to its rest energy in reference frame Σ :

- increased by the additional energy given to it in frame of reference Σ if the action on the particle takes place in frame of reference Σ, then: $m'_0 = \Upsilon m_0$
- reduced by the energy decrement it undergoes in frame of reference Σ' if the action on the particle is exerted in frame of reference Σ', then: $m'_0 = m_0 / \Upsilon$

In fact, depending on the action exerted, m'_0 can take any value between m_0 / Υ and Υm_0 . We can say that the rest energy of a particle depends on its history.

Let us now observe that an experiment must necessarily be defined in a reference frame where we know the rest energy of the particles and where we can define the actions exerted. The energies involved are exchanged within this frame of reference. Let's call this frame of reference Σ.[16]

The transition to the reference frame Σ' in motion is made by adopting a rest mass $m'_0 = \Upsilon m_0$ (the action to bring the particle to rest in Σ' takes place in Σ)[17] . Equation (3.8) becomes:

$$E' = E\,(1 - \underline{u}\,\underline{v})/(1 - \underline{u}^2) \qquad (3.13)$$

And the inverse equation (3.8') gives:

$$E = E'\,(1 + \underline{u}\,\underline{v}') \qquad (3.13')$$

Equation (3.13) can also be written as:

$$E' = \Upsilon^2\,(E - \underline{u}\,p\,c) \qquad (3.14)$$

Looking on the equations for changing coordinates, we can see straight away that Lorentz's formulae are no longer respected.

With $m'_0 = \Upsilon m_0$ equation (3.12) becomes:

$$dt' = \Upsilon^2\,(dt - (u/\mathbf{c}^2)\,dx) \qquad (3.15)$$

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[15] The term "kinetic energy" becomes inappropriate. We have to consider that the acceleration imparted to the particle has two effects: it increases its momentum and it increases its total energy. How does this increase in stored energy affect the particle? Its lifetime is increased (see paragraph 3.3.1). We will also see in Chapter 5 (Gravitation) that, when a gravitational source is in motion, it is its total energy that must be taken into account to determine its gravitational action, just as it is the relativistic mass that comes into play in the fundamental equation of dynamics.

[16] Here we return to what was said in Chapter 2 about events. The experiment is considered to be "produced" in frame of reference Σ.

[17] If, for an experiment carried out in Σ, we know the rest mass m'_0 in Σ', the rest mass to be retained in Σ is obviously: $m_0 = m'_0 / \Upsilon$.

The coordinate change equations can then be written as :

$$x' = \Upsilon^2 (x - u\, t) \qquad (3.16)$$
$$t' = \Upsilon^2 (t - u\, x/\mathbf{c}^2)$$ identical to equations (2.8)

or, taking the inverse equations:

$$x = x' + u\, t' \qquad (3.16')$$
$$t = t' + u\, x'/\mathbf{c}^2$$

If we started with an experiment in Σ', the equations would be :

$$x = \Upsilon^2 (x' + u\, t') \qquad (3.16a)$$
$$t = \Upsilon^2 (t' + u\, x'/\mathbf{c}^2)$$ identical to equations (2.8a)

A single set of coordinates [x',t'] in reference frame Σ' can therefore correspond to two distinct sets in reference frame Σ :

- by the equations (3.16'): [x, t]
- by equations (3.16a): [Υ^2 x, Υ^2 t]

We explained in sub-chapter 2.4 why there is no contradiction.

3.3. Consequences of total energy invariance

3.3.1. Influence of energy on clock rhythm

We saw in paragraph 2.3.4 that a moving (atomic) clock lags behind a clock that remains stationary in a reference frame. With our hypothesis [18] **, this can only be explained by admitting that the energy level influences the rhythm of the clock.** [19]

If T is the period of a clock at rest and T' the period of the same clock set in motion at velocity u, then:

$$T'/T = E'_0/E_0 = m'_0/m_0 = \Upsilon = 1/(1- u^2/\mathbf{c}^2)^{1/2} \qquad (3.17)$$

Note that if Δt and Δt' are the durations measured by the clocks, then:

$$E_0\, \Delta t = E'_0\, \Delta t' \qquad (3.18)$$

The product of energy and time measured by the clock is an invariant.

[18] See § 2.4.2.
[19] We also propose a simple reasoning explaining this influence in the case of an atomic clock using caesium atoms.

Depending on the choice of invariant made for energy, we are therefore led to different interpretations for certain phenomena considered as the basis for experimental verification of the theory of special relativity:

- In the RR framework, the delay of moving clocks and the increase in the lifetime of atmospheric muons travelling at high speed are attributed to the dilation of time between reference frames; it is assumed that the clocks continue to deliver the same unit of time;

- with the choice of conservation of total energy by change of reference frame, these phenomena are attributed to the modification of the rest energy of the atoms of the atomic clock or that of the muons, due to the energy received to pass from the reference frame Σ to the reference frame Σ'; the flow of time is not modified, but the clocks no longer deliver the same unit of time.

Can this influence of energy on the rhythm of clocks be extended to types of clocks other than atomic clocks?

Mechanical clocks should not be affected insofar as all their constituent elements undergo the same relative variation in mass. For example, a simple oscillator of mass m_0 and stiffness k is characterised by a period $T = 2\pi\,(m_0\,/k)^{1/2}$ which remains unchanged if the stiffness varies with the mass.

A light pulse clock can be schematised as a tube closed by two parallel mirrors between which photons travel back and forth, the number of paths is counted by a mechanism. The universality of the speed of light[20] means that such a clock is not affected by being set in motion at rectilinear and uniform velocity.

3.3.2. Concept of space and time

Special relativity can be associated with the conception of a space-time considered as an affine space of dimension 4. In the vector space associated with this affine space, time differs from space coordinates in the definition of the scalar product, which is not Euclidean: the product of time-type coordinates is assigned the opposite sign to that of the products of space-type coordinates.

This leads us to define the square of the space-time interval between two events as :

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - \mathbf{c}^2\,\Delta t^2 \qquad (3.19)$$

In special relativity, this quantity is invariant to any change of Galilean reference frame. It is easy to show that Lorentz's formulae can be deduced from this invariance.

The elegance of this mathematical formulation undoubtedly contributed greatly to the acceptance of the theory, despite the counter-intuitive nature of a different flow of time in two reference frames in relative motion and the contradiction that results.

In our proposed approach, we reject this conception of space and time. The flow of time remains universal and completely independent of the reference frame, as in Newtonian mechanics.

[20] See § 4.1.2.

On the other hand, time is not absolute because of the existence of a limiting speed, which is identical in all frames of reference because of the principle of relativity: the clocks in the moving frame of reference appear to be out of sync with those in the frame of reference considered to be fixed.

We have shown that the solution obtained for the reference frame change equations is consistent with the (Einsteinian) hypothesis of conservation of the total energy of a particle, in place of the invariance of the rest mass.

The spatial distance between two events remains invariant in all Galilean reference frames, as in Newtonian Mechanics.

3.3.3. Preferred reference frame, true velocity and true energy

Equation (3.13) shows that E' is different from E, except for $v = u$.

The conservation rule for total energy means that, in any reference frame, the energy of a particle at rest is "true" energy, representing the maximum amount of energy that can be released.[21] On the other hand, the total energy attributed to a moving particle is not necessarily "true" energy.

The reference frame in which an experiment is defined plays a key role: the energy at rest is known, as are the actual energy transfers. The energy calculated in this frame of reference is "true" energy at all times. This property can be made the distinguishing feature of this privileged frame of reference compared with other frames of reference.

The velocity of the particle must therefore also be considered as a "true" velocity in the reference frame of the experiment.

As a function of velocity and total energy, the fundamental equation of dynamics is written:

$$(v/c)\, d(E\, v/c) = dE$$

In the reference frame of the experiment, this equation involves a "true" velocity and a "true" energy, whereas these are apparent values of these quantities in another reference frame.

[21] But, as we saw in paragraph 3.2.2, two identical particles at rest in the same frame of reference can have different energies, depending on the actions they have undergone.

4. *Taking account of light phenomena*

4.1. Speed and frequency of electromagnetic waves

4.1.1. Speed of light as the limiting speed

Light is electromagnetic radiation. According to Maxwell's electromagnetic theory, this radiation can be represented as an electromagnetic wave, which propagates uniformly in all directions from its source, its speed $\mathbf{c}$ being related to two constants :

- the dielectric permittivity of vacuum: $\boldsymbol{\varepsilon_0}$
- the magnetic permeability of vacuum: $\boldsymbol{\mu_0}$

by the relation: $\mathbf{c}^2 = 1 / \boldsymbol{\varepsilon} \boldsymbol{\mu_0}$

It seems logical to assume that $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ are universal constants, having the same value whatever the Galilean reference frame considered, and therefore that the value of **c does** not depend on the reference frame either: **light emitted by a fixed source in a Galilean reference frame propagates at speed c in this reference frame.**

On the other hand, electromagnetic theory predicts that a flash of light containing energy E must have impulse[22] $\mathrm{E}/\mathbf{c}$, which is verified by experiment. It follows that equation (3.6) linking energy and momentum remains valid for a flash of light, provided that its energy and momentum are carried by particles of zero mass: photons.

Equation (3.5) shows that it is possible to associate a finite energy with a particle of infinitely small mass when its speed tends towards the limiting speed $\mathbf{c}$.

The speed of light is therefore the limiting speed for massive particles. This is verified by experiment. The coordinate equations (2.7) and (3.16) are therefore equally applicable to the propagation of light waves.

4.1.2. Universality of the speed of light

In the theory of special relativity, the invariance of the space-time interval (cf. equation 3.9) leads immediately to the following result: **the speed of light is equal to c in all reference frames, whether the source is fixed or moving and whatever the direction of propagation.**

In the alternative approach based on the conservation of total energy, the emission of a light beam must necessarily be related to the reference frame where its source is fixed and its speed equal to $\mathbf{c}$. The invariance of distance and duration in all reference frames[23] means that the "true" velocity of light remains equal to $\mathbf{c}$, whatever the reference frame used for calculation.

[22] The impulse of a photon is the momentum it gives up when it is absorbed.
[23] c. § 3.3.2.

On the other hand, the apparent velocity of light is less than $\mathbf{c}$ in directions other than that of the displacement of the moving frame of reference (also equal to $\mathbf{c}$ for propagation perpendicular to that of the displacement of the frame of reference). This is easily verified by checking that the expression ($\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - \mathbf{c}^2 \Delta t'^2$) is negative when ($\Delta x^2 + \Delta y^2 + \Delta z^2 - \mathbf{c}^2 \Delta t^2$) is zero.

4. 1.3. Doppler effect

As mentioned above, according to Maxwell's theory, the propagation of a light wave is defined in relation to its source; we need to reason in the frame of reference where this source is fixed.

When an observer moves relative to a light source, the frequency he perceives changes. Calculated in the reference frame of a source of natural frequency ν_S[24] , this frequency is :

$\nu = (1 - \mathbf{u/c})\, \nu_S$ if the observer moves away from the source at velocity $u>0$; (4.1)

$\nu = (1 + \mathbf{u/c})\, \nu_S$ if the observer moves closer to the source.

In the observer's frame of reference, we have to take into account :

- *in the case of special relativity*, the dilation of durations; the periods measured are reduced, the frequencies increased, they become :

  $$\nu = \Upsilon(1 - \mathbf{u/c})\, \nu_S = ((1 - \mathbf{u/c})/(1 + \mathbf{u/c}))^{1/2}\, \nu_S \quad (4.2)$$

  and $$\nu = \Upsilon(1 + \mathbf{u/c})\, \nu_S = ((1 + \mathbf{u/c})/(1 - \mathbf{u/c}))^{1/2}\, \nu_S$$

- *in the case of conservation of total energy*, equations (4.1) remain valid if the observer's clock delivers the same unit of time as that of the source.[25] In the terrestrial experiments that we can carry out, this is not the case: the observer is set in motion relative to the source, or the source is set in motion relative to the observer.
  In the first hypothesis, the observer's clock beats at a slower rate and the apparent frequency of the wave increases; we find equations (4.2). In the second case, the same equations apply, but the source, which has seen its energy increase by a factor of Υ, sees its natural frequency increase by the same factor to $\Upsilon\, \nu_S$.[26]

Equations (4.2) are verified by experiments.

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[24] The natural frequency is the frequency of the source in the reference frame where it is at rest.
[25] The transition from the reference frame of the source to that of the observer takes place without any time dilation (cf. § 3.3.1.).
The result can be verified from the wave equation in the reference frame where the source is fixed.
This equation is written : $A = A_0 \sin(\omega(\mathbf{t\text{-}x/c}))$ for a wave of speed + $\mathbf{c}$.
The change of coordinates gives (equations 3.16'): $A = A_0 \sin(\omega(t' + u\,x'/\mathbf{c}^2 - x'/c - u\,t'/\mathbf{c}))$
u being the observer's velocity relative to the source.
or : $A = A_0 \sin(\omega(1 - \mathbf{u/c})(t' - x'/\mathbf{c}))$
[26] see 4.2. below; photon energy is proportional to frequency.

4.1.4. Interpretation of the spectral shift of distant sources

In the context of special relativity, it is easy to relate the frequency observed in a terrestrial reference frame to the velocity of the source, since the frequency at rest is assumed to be identical in all reference frames: all we have to do is apply equations (4.2).

In the case of the conservation of total energy, the frequency of the source is not known a priori because, for the same element, the rest mass can be different from one reference frame to another. If we assume that the natural frequency of the source is the same as in the terrestrial reference frame, the equations (4.1) apply; the result is therefore different from that given by the theory of special relativity.

However, the difference only becomes significant when the $\mathbf{v}/\mathbf{c}$ ratio can no longer be considered small compared to 1.

4.2. Energy associated with an electromagnetic wave

The corpuscular representation of electromagnetic radiation consists of considering it as made up of photons, each carrying a quantum of energy.

4.2.1. Energy of a photon

In the frame of reference where the source is fixed, the quantum of energy of a photon is given by the Planck-Einstein relation :

$$E_S = \mathbf{h}\, \nu_S$$

$\mathbf{h}$ being Planck's constant and ν_S the frequency of the wave associated with the photon.

This is the "true" energy of the photon.

How does the energy of a photon vary when the reference frame changes?

We are going to check that the equations in sub-chapter 3.2 remain valid for particles of zero mass. This condition is necessary to satisfy the principle of relativity in the case of photons interacting with matter.

Consider the case of a particle initially at rest in Σ, of energy E_0, having absorbed a photon of energy E_S emitted in the same frame of reference :

- its energy is: $E = E_0 + E_S$;
- its momentum is equal to the impulse of the absorbed photon:

  $E\, v/\mathbf{c}^2 = \varepsilon\, E_S\, /\mathbf{c}$ where: $\varepsilon = \pm 1$ depending on whether the photon has a speed $\pm\, \mathbf{c}$ **;**
- its speed is therefore such that: $v/\mathbf{c} = \varepsilon\, E_S / E$.

Let u be the velocity of the moving frame of reference Σ' relative to the source. Equations (3.9) and (3.13) giving the energy E' in Σ' as a function of E can be written :

$E' = a\,(1 - u\,v/\mathbf{c}^2)\,E$ where: $a = \Upsilon$ (assuming invariance of rest mass) (4.3)

$a = \Upsilon^2$ (invariance of total energy, experiment in Σ)

Given the expression for v given above, E' can be written as :

$$E' = a\,(1 - u\,\varepsilon\,E_S\,/\,\mathbf{E/c})\,E = a\,(E - \varepsilon\,u\,E_S\,/\mathbf{c})$$

If we denote by E'_φ the energy of the photon in Σ', E' can also be written as :

$$E' = a\,E_0 + E'_\varphi = a\,(E - E_S\,) + E'_\varphi$$

Therefore : $a\,(E - \varepsilon\,u\,E_S\,/\mathbf{c}) = a\,(E - E_S\,) + E'_\varphi$

From this we derive : $E'_\varphi = a\,(1 - \varepsilon\,\mathbf{u/c})\,E_S$ (4.4)

We find equation (4.3) with $v = \varepsilon\,\mathbf{c}$.

a) <u>In special relativity</u>, equation (4.4) leads to :

for $v = \mathbf{c}$: $E'_\varphi\,/E_S = ((1 - \mathbf{u/c})/(1 + \mathbf{u/c}))^{1/2}$ (4.5)

for $v = -\,\mathbf{c}$: $E'_\varphi\,/E_S = ((1 + \mathbf{u/c})/(1 - \mathbf{u/c}))^{1/2}$

The Doppler effect relationships (4.2), linking apparent and natural frequencies, are used again.

For the observer in reference frame Σ', the photon has an apparent energy $\mathbf{h}\,\nu$ corresponding to its apparent frequency.

b) <u>Assuming the conservation of total energy</u>, <u>for an experiment carried out in reference frame Σ where the source is fixed</u> ($a = \Upsilon^2$):

$E'_\varphi = ((1 - \varepsilon\,\mathbf{u/c})\,(1 - u^2\,/\mathbf{c}^2\,))\,E_S = E_S/(1 + \varepsilon\,\mathbf{u/c})$ (4.6)

for $v = \mathbf{c}$: $E'_\varphi\,/E_S = 1/(1+ u/c)$

for $v = -\,\mathbf{c}$: $E'_\varphi\,/E_S = 1/(1- \mathbf{u/c})$

These relationships are not identical to the relationships (4.1) relating to frequencies:

for $v = \mathbf{c}$: the observer moves away from the source and $\nu = (1 - \mathbf{u/c})\,\nu_S$

for $v = -\,\mathbf{c}$: the observer moves closer to the source and $\nu = (1 + \mathbf{u/c})\,\nu_S$

The apparent energy of the photon in Σ' is equal to $\Upsilon^2\,\mathbf{h}\,\nu$.

c) What about an experiment carried out in reference frame Σ' where the source is mobile?

The same reasoning as above leads to the relationship below (obtained by replacing u by - u in (4.6)):

$$E_S = \Upsilon^2 (1 + \varepsilon\, \mathbf{u/c})\, E'_\varphi = E'_\varphi / (1 - \varepsilon\, u/\mathbf{c})$$

or :

$$E'_\varphi = (1 - \varepsilon\, \mathbf{u/c})\, E_S \qquad (4.8)$$

This relationship is identical to that giving the apparent frequency. The photon therefore has an apparent energy in Σ' equal to $\mathbf{h}\,\nu$.

Whether the source is fixed or moving, the energy of the photon is therefore given by the product $\mathbf{h}\,\nu$ in the reference frame of the experiment.

4.2.2. Power of an electromagnetic wave

An electromagnetic wave can be considered as a flow of photons. But, conversely, does any flow of photons constitute an electromagnetic wave?

We don't think so. The frequency of the wave not only defines the energy of the photons, but also their sequence in time. A photon can therefore be considered as a fragment of an electromagnetic wave (of frequency ν_1), but a succession of photons (at frequency $\nu_2 \neq \nu_1$) is just a succession of wave fragments.

What consequences does this have for the power of a wave?

Let's imagine the following experiment:

A monochromatic wave of frequency ν carries a flux of N photons per second across a surface S; its power is equal to : $N\, \mathbf{h}\, \boldsymbol{\nu} / S$.

The photon flux results from both the succession and the superposition of photons. If we keep the superposition unchanged, transforming this wave into a wave of frequency ν' means changing the photon flow rate to (ν' / ν) N photons per second.

The power becomes : $(\nu' / \nu\, N)\, \mathbf{h}\, \nu' / S = (\nu'^2 / \nu^2)\, (N\, \mathbf{h}\, \nu / S)$

We must therefore consider that the power of an electromagnetic wave varies as the square of the frequency.

We will see in the next chapter that electromagnetic waves can be distorted by a gravitational field. The emission frequency and the energy-related frequency can then be different.

5. Gravitation

5.1. Formulation of the laws of gravitation

The theory of general relativity retains the principle of invariance of mass at rest. It also retains the equality between gravitational mass and inertial mass and establishes the principle of equivalence between gravitation and acceleration.

In our proposal, we retain the equality between gravitational mass and inertial mass. However, we will see that gravitation and acceleration cannot be considered to be completely equivalent.

An initial analysis based on experimental observations (Newtonian law of gravitation, gravitational shift of clocks) will enable us to establish the laws of gravitation in conditions of weak gravitational field and to identify the principles that we propose to retain.

5.1.1. Weak gravitational field laws

Let's consider a massive body of mass M, constituting the gravitational source. In the reference frame (assumed to be Galilean) linked to this body, a particle [27] located at distance r has a non-zero rest energy E_0 (rest mass m_0).

Throughout this chapter we shall consider that M *is very large compared with* m_0 *so that we can regard* M *as invariant and therefore neglect the influence of* m_0 *on* M*. We shall also assume that the field is spherically symmetrical.*

We assume that the action of the massive body on the particle can be described using a gravitational field which gives the particle a potential energy E_g. [28]

Throughout Chapter 5, we will assume that the ratio GM/c^2 ***r is small compared to*** 1 **(weak field).**[29] **We assume that, in this case, the Newtonian law of gravitation is valid at zero speed.**

Energy equations

Let's exert an action on the particle at rest so as to move it away (at infinitesimal speed) by a distance dr in a radial direction with respect to the source.

[27] Remember that, in this note, the term particle applies to any physical object that can be modelled as a point to describe its motion.
[28] This proposal is justified in the note entitled "Gravitational field, Fundamental Principle of Dynamics and Quantum Mechanics" setting out the model chosen for the field.
[29] G is the universal gravitational constant.

How does the particle's energy vary?

According to the Newtonian law of gravitation (G being the universal gravitational constant), the force required to move the particle is just greater than :

$$F_x = GM\, m_0 / r^2 \qquad (5.1)$$

For an incremental displacement dr, the external energy supplied is equal to the work of the force :

$$dE_x = (GM\, m_0 / r^2)\, dr = (GM/ \mathbf{c}^2 r^2)\, E_0\, dr$$

Following on from what was developed in paragraph 3.2.2, we extend the questioning of the invariance of mass at rest by considering that, if a particle is subject to the gravitational influence of a massive body, its energy at rest (in the reference frame of the massive body) varies with its distance from this body.

This variation in resting energy is consistent with the gravitational offset of the clocks: clocks on the ground deliver shorter durations than clocks on board satellites. In line with what was explained in paragraph 3.3.1, this can be interpreted as linked to an increase in the resting energy due to the increase in the gravitational effect. So, in accordance with the experimental results, the variation in energy is expressed by :

$$dE_0 = -\, (GM/ \mathbf{c}^2 r^2)\, E_0\, dr \qquad (5.2)$$

This assumption of variation in rest energy in the gravitational field (in the same reference frame) implies that the effect of gravitation cannot be completely assimilated to that of acceleration.

The conservation of the overall energy of the system implies that the variation in potential energy is equal to:

$$dE_{g0} = dE_x - dE_0 = 2\, (GM/ \mathbf{c}^2 r^2)\, E_0\, dr$$

This formulation logically leads us to extend the result by replacing E_0 by E (total energy of the particle) when the particle is in motion:

$$dE_g = dE_x - dE = 2\, (GM/ \mathbf{c}^2 r^2)\, E\, dr \qquad (5.3)$$

In so doing, we consider that gravitation is a phenomenon of energy exchange between the gravitational field and the particle, involving the latter's total energy.

Finally, the energy equations governing the motion of the particle in the gravitational field can be written incrementally as follows:

$$dE_0 = -\, (GM/ \mathbf{c}^2 r^2)\, E_0\, dr \qquad (5.2)$$

$$dE = dE_x - 2\, (GM/ \mathbf{c}^2 r^2)\, E\, dr \qquad (5.4)$$

These equations are not contradictory because, *in the absence of external forces,* the particle cannot be at rest and we necessarily have: $E \neq E_0$.

By integration, equations (5.2) and (5.4) lead to the relationships below: [30]

- for energy at rest :

$$E_0 = E_{0\infty} \exp(GM/\mathbf{c}^2 r) \approx (1 + GM/\mathbf{c}^2 r)\, E_{0\infty} \qquad (5.5)$$

- for total energy (*in the absence of external forces*) :

$$E = E_\infty \exp(2\,GM/\mathbf{c}^2 r) \approx (1 + 2\,GM/\mathbf{c}^2 r)\, E_\infty \qquad (5.6)$$

Gravitational potential

By choosing $E_g = 0$ at infinity, *in the absence of external forces*, the potential energy is:

$$E_g = E_\infty - E \approx -(2\,GM/\mathbf{c}^2 r)\, E_\infty \qquad (5.7)$$

Assuming $m_{0\infty}$ = rest mass excluding the influence of gravity: $E_\infty = \Upsilon_\infty\, m_{0\infty}\, \mathbf{c}^2$

Therefore : $E_g \approx -2\,\Upsilon_\infty\, GM\, m_{0\infty} / r$

For zero speed at infinity, the potential is double that derived from Newtonian theory. This is due to the effect of gravitation on the rest mass. For a non-zero speed at infinity, the deviation from the Newtonian potential increases with the speed.

Equation of motion

The effect of gravitation can be considered as that of an acceleration applied step by step to a variable mass at rest. As the potential energy is proportional to the energy of the particle, the motion is independent of the initial mass if the initial velocities are identical.

The fundamental equation of dynamics can be used, bearing in mind that the change in energy associated with the change in momentum (i.e. dE_p) does not represent the entire change in energy dE of the particle.

Part of the variation in potential energy corresponds to the variation in the particle's rest mass, i.e. :

ΥdE_0 (rest mass of energy dE_0 carried at speed v).

So we have : $dE_p = dE - \Upsilon\, dE_0$

The effect of gravitation on momentum is likened to that of an attractive force $\vec{F}$ collinear with $\vec{r}$:

$$d(m\vec{v}) = (\vec{F} + \overrightarrow{F_x})\, dt$$

$$\vec{v}\, d(m\vec{v}) = \vec{v}\,(\vec{F} + \overrightarrow{F_x})\, dt = \vec{F}\,\overrightarrow{dr} + dE_x = dE_p$$

Since : $\vec{v}\,\vec{v} = v^2$, $v^2\, dm + m\,\vec{v}\, d\vec{v} = dE_p$

[30] The introduction (for convenience) of exponential functions in no way prejudges the extension of the laws of gravitation outside the weak field domain.

In this equation, m corresponds to the total energy of the particle ($m = \mathbf{E}/\mathbf{c}^2$), while dm is related to the variation in momentum ($dm = dE_p / \mathbf{c}^2$). Therefore :

$$(v^2 / \mathbf{c}^2)\, dE_p + (\vec{v}\, d\vec{v} / \mathbf{c}^2)\, E = dE_p \qquad (5.8)$$

$\Upsilon^2 = 1/(1 - v^2/\mathbf{c}^2)$ $\rightarrow$ $v^2 = \mathbf{c}^2 (1 - 1/\Upsilon^2)$ and $\vec{v}\, d\vec{v} = \mathbf{c}^2\, d\Upsilon / \Upsilon^3$

(5.8) $\rightarrow$ $(1 - 1/\Upsilon^2)\, dE_p + (d\Upsilon/\Upsilon^3)\, E = dE_p$

$$- dE_p + (d\Upsilon/\Upsilon)\, E = 0$$

Therefore : $dE_p = (d\Upsilon/\Upsilon)\, E$ (5.9)

This means that Υ is a solution to the equation :

$$d\Upsilon/\Upsilon = dE/E - \Upsilon\, dE_0 / E$$

It is easy to see that the choice $\Upsilon = E/E_0$ is appropriate. This shows that the assumption that the rest energy varies is consistent with the concept of total energy.

The above equation can then be written as :

$$d\Upsilon/\Upsilon = dE/E - dE_0 / E_0$$

which corresponds to the derivation of the equation : $\Upsilon = E/E_0$.

Finally : $Fdr = dE - \Upsilon\, dE_0 - dE_x = (dE - dE_x) - (dE_0 / E_0)\, E$

$$\approx -2\, (GM/\mathbf{c}^2 r^2)\, E\, dr + (GM/\mathbf{c}^2 r^2)\, E\, dr$$

$$F \approx - GM\, E/\mathbf{c}^2 r^2 = - \Upsilon\, GM\, E_0 / \mathbf{c}^2 r^2 \qquad (5.10)$$

For $\Upsilon = 1$ we find the Newtonian expression.

Since $\Upsilon = E/E_0$, we deduce from equations 5.5 and 5.6 (*in the absence of external forces*):

$$\Upsilon = \Upsilon_\infty \exp(GM/\mathbf{c}^2 r) \approx \Upsilon_\infty (1 + GM/\mathbf{c}^2 r) \qquad (5.11)$$

Sharing the variation in potential energy

It's easy to check that :

$$\Upsilon\, dE_0 = d\Upsilon\, E_0 \qquad [= -\Upsilon_\infty (GM/\mathbf{c}^2 r^2) \exp(GM/\mathbf{c}^2 r)\, E_0\, dr]$$

Consequently, the energy exchanged with the gravitational field is shared equally between the acceleration effect and the effect of modifying the energy at rest.

We will retain the above conclusion as a principle and show that equations (5.1) and (5.2) can be derived from it.

5.1.2. Principles adopted

According to the preceding analysis, we adopt the following principles to establish the laws of gravitation for a non-zero mass particle in a weak field:

First principle: gravitational field and potential energy

The gravitation created by a gravitational source can be characterised by a gravitational field that imparts potential energy to the particles present in it.

The variation in the potential energy of a particle, of zero or non-zero mass, is proportional to its total energy E, calculated in the reference frame linked to the source; it is given by the relation :

$$dE_g = 2\ (GM/\ \mathbf{c}^2r^2)\ E\ dr$$

Second principle: conservation of energy

The variation in total energy of a particle is equal to the opposite of the variation in potential energy plus the work of external forces, if any:

$$dE = dE_x - dE_g$$

Third principle: the influence of gravitation on a particle's energy and momentum

The rest energy of a non-zero mass particle varies with its distance from the source.

In the absence of external forces, the variation in energy associated with rest energy and the variation in energy associated with momentum are each equal to and opposite half the variation in potential energy.

Fourth principle: (partial) equivalence between gravitation and acceleration

The fundamental law of dynamics can be applied to determine the relationship between the variation in the particle's momentum and the variation in energy associated with this momentum under the effect of the gravitational field.

Application of the principles

Let's consider the absence of external forces:

$1^{st} + 2^{nd}$ principle: $dE = -dE_g = -2\ (GM/\ \mathbf{c}^2r^2)\ E\ dr$

3^{rd} principle: $\Upsilon\ dE_0 = dE/2$

4^{th} principle: $E = \Upsilon\ E_0$ (see ***Equation of motion*** in paragraph 5.1.1)

It follows immediately :

$$dE = 2\ \Upsilon\ dE_0 = -2\ (GM/\ \mathbf{c}^2r^2)\ \Upsilon\ E_0\ \ dr$$

$$dE_0 = - (GM/ \mathbf{c}^2 r^2)\, E_0 \;\; dr \qquad \text{(i.e. equation 5.2)}$$

Referring to the ***equation of motion,*** we see that :

$$dE_p = dE - \Upsilon\, dE_0 = F\, dr$$

Therefore : $F\, dr = - (GM/ \mathbf{c}^2 r^2)\, E\, dr$

If F_x is the opposite of the attractive gravitational force and the speed is zero:

$$F_x = GM\, E_0 \,/\, \mathbf{c}^2 r^2 = GM\, m_0 \,/ r^2 \qquad \text{(i.e. equation 5.1)}$$

5.1.3. The case of photons. Gravitational spectral shift

When applied to photons, equation (5.4) on total energy explains :

- the deflection of light rays by a gravitational source (see sub-chapter 5.3);
- the gravitational spectral shift (see sub-chapter 5.4 : Pound and Rebka experiment).

In section 4.2.2 we saw that the frequency of an electromagnetic wave represents both the emission frequency of photons and the frequency linked to their energy by the Planck-Einstein relation.

In accordance with this relationship, the energy-related frequency follows the same law as the total energy:

$$d\nu = - 2\, GM/ \mathbf{c}^2 r^2\, \nu\, dr \qquad (5.12)$$

In a gravitational field, the frequency related to the energy of a photon varies; it is given by :

$$\nu_g = \nu_\infty \exp\, (2\, GM/ \mathbf{c}^2 r) \approx (1 + 2\, GM/ \mathbf{c}^2 r)\, \nu_\infty \qquad (5.12')$$

On the other hand, the frequency ν_0 at which the wave is emitted (which defines the succession of photons) is necessarily invariant ($= \nu_\infty$), otherwise there would be an accumulation of photons.

The energy frequency and the emission frequency therefore become distinct.

Equation (5.5) on the rest energy disappears since the photon has zero rest mass.

However, at the same time as the spectral shift, the speed of propagation c_g of a photon is modified as it passes through the gravitational field. [31]

In a gravitational field, the wavelength λ_g of an electromagnetic wave is expressed by the dual relation:

$$\lambda_g = \mathbf{c} / \nu_g = c_g / \nu_0$$

[31] This explains the Shapiro effect (Influence of a gravitational field on the transmission time of an electromagnetic signal) (see § 5.5).

The first term in this relationship corresponds to the "energetic" calculation of the wavelength :

$$\mathbf{c} / \nu_g = h/p$$

since : E (energy) = $\mathbf{h}\, \nu_g$ and p (impulse) = $E / \mathbf{c}$ [32]

The second term corresponds to the kinematic calculation (celerity divided by the emission frequency).

The products $\mathbf{c}_g\, \nu_g$ and $\lambda_g\, \nu_g$ are invariant across the field.[33]

The speed of the wave (and photon) is :

$$c_g = \exp(-2\, GM/ \mathbf{c}^2\, r) \approx (1 - (2\, GM/ \mathbf{c}^2\, r)\, \mathbf{c} \qquad (5.13)$$

The above leads us to complete the principles set out earlier by writing that, when particles move in a gravitational field, the action of gravitation consists of :

- a change in the total energy of the particles, whether or not they have zero mass ;
- a change in the momentum of non-zero mass particles and their rest energy;
- a change in the momentum of zero-mass particles and their speed, which becomes less than **c**.

Comparison with General Relativity

Given equation (5.12'), the equation linking the frequencies at two points, at distances from the gravitational source r_1 and r_2 , is written :

$$\nu_2 = \nu_1\, (1 + 2\, GM/ \mathbf{c}^2 r_2)/ (1 + 2\, GM/ \mathbf{c}^2 r_1) \qquad (5.14)$$

In the context of general relativity, the equation obtained is :

(GR) $$\nu_2 = \nu_1\, ((1 - 2\, GM/ \mathbf{c}^2 r_1)/ (1 - 2\, GM/ \mathbf{c}^2 r_2))^{1/2}$$

or, in low-field conditions :

(GR) $$\nu_2 = \nu_1\, ((1 + GM/ \mathbf{c}^2 r_2)/ (1 + GM/ \mathbf{c}^2 r_1)) \qquad (5.14')$$

The frequency difference is therefore twice as small for general relativity.

[32] We consider that the impulse remains equal to $E/\mathbf{c}$ even though the speed of the wave is no longer equal to **c**.

[33] We can also explain this result in the following way: as mentioned in paragraph 4.2.2, if the speed of the wave remained equal to **c**, its power would vary as the square of its frequency; this would imply that, as the wave progressed towards the gravitational source, the number of photons elapsed per unit time would increase, which cannot be the case.

Let us now consider a photon source located at distance r_1 , emitting photons of frequency ν_{S1} . The same photon source located at distance r_2 would emit photons of frequency ν_{S2} , given by equation :

$$\nu_{S2} = \nu_{S1} \, (1 + GM/\,\mathbf{c}^2 r_2\,)/\,(1 + GM/\,\mathbf{c}^2 r_1\,) \qquad (5.15)$$

This is because the energy of the sources, and therefore that of the photons emitted, is shifted in accordance with equation (5.5) concerning energies at rest.

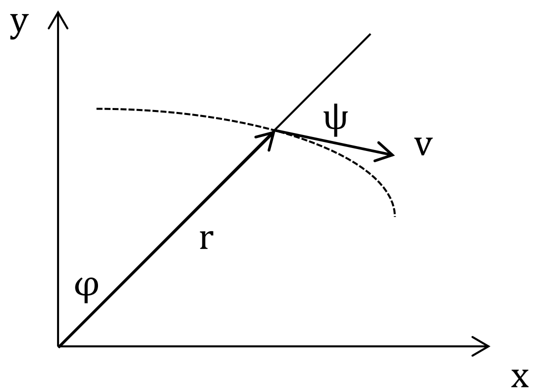
The equation giving the frequency ν_2 at r_2 of the photon emitted at r_1 as a function of the frequency ν_{S2} is as follows:

$$\nu_2 = \nu_{S2} \, (1 + GM/\,\mathbf{c}^2\, r_2)/\,(1 + GM/\,\mathbf{c}^2\, r_1)$$

We can see that this equation is identical to equation (5.14') given by General Relativity (which accepts $\nu_{S2} = \nu_{S1}$).

This reasoning will be used to explain the result of the Pound and Rebka experiment (see § 5.4.).

5.1.4. Case of the action of the gravitational field alone. The theorem of angular momentum



Let's consider a plane motion, which is the case for planetary orbits. If the particle is not subject to any force other than the centripetal force linked to the gravitational field, we can solve the problem using the theorem of angular momentum.

Let us denote by $\vec{p} = m\,\vec{v}$ the momentum. According to the fundamental law of dynamics $d\,\vec{p}$ and $\vec{r}$ are collinear. On the other hand $\vec{v}$ and $d\vec{r}$ are also collinear (since : $\vec{v} = d\vec{r}/dt$).

Therefore : $d\,(\vec{p} \wedge \vec{r}\,) = d\,\vec{p} \wedge \vec{r} + \vec{p} \wedge d\vec{r} = 0$

from which it follows : $m\,v\,r\,\sin(\vec{v},\vec{r})$ = Constant (5.16)

Equations (5.2) and (5.4) give v as a function of r and equation (5.16) gives the angle Ψ of $\vec{v}$ and of $\vec{r}$.

This result is applied below to the determination of Mercury's perihelion advance.

It can also be applied to the calculation of the curvature of light rays in a gravitational field, assuming that the variation in photon impulse is directed towards the source of the field (cf. § 5.3).

5.2. Calculating Mercury's perihelion advance

5.2.1. The polar coordinates of the planet in a reference frame related to the sun are (r, φ).

The radial and angular speeds are :

$$dr/dt = v \cos\psi \qquad d\varphi/dt = (v/r)\sin\psi$$

(5.16) → $\Upsilon\, m_0\, v\, r \sin\psi = A$ (constant) (5.16')

Starting from ϕ = 0 when the planet is at perihelion, we look for the value of φ when it reaches aphelion. Using the above relationships, we can write :

$$\varphi = \int (d\varphi/dt)\, dt = A\int dt/(\Upsilon\, m_0 r^2) = A\int (dr/(\Upsilon\, m_0 r^2))(dt/dr) = A\int (dr/(\Upsilon\, m_0 r^2))/(v\cos\psi)$$

$$(v\sin\psi)^2 = (A/(\Upsilon\, m_0\, r))^2 \quad\rightarrow\quad v\cos\psi = (v^2 - (A/(\Upsilon\, m_0\, r))^2)^{1/2}$$

And finally : $\varphi = A\int (dr/r^2)/((\Upsilon\, m_0 v)^2 - (A/r)^2)^{1/2}$

$$\varphi = -\int d(1/r)/((\Upsilon\, m_0 v/A)^2 - (1/r)^2)^{1/2} \qquad (5.17)$$

5.2.2. Equation (5.11) can also be written:

$$\ln\Upsilon = GM/\mathbf{c}^2 r + \text{constant} \qquad (5.18)$$

*For Mercury the **v/c** ratio varies from 1.3 10^{-4} to 2 10^{-4} between aphelion and perihelion. We will continue the calculation using limited developments of order 2 in v/**c** :*

$$\Upsilon = 1 + (v/\mathbf{c})^2/2 + 3\,(v/\mathbf{c})^4/8$$

$$\ln\Upsilon = (v/\mathbf{c})^2/2 + (v/\mathbf{c})^4/4$$

(5.18) → $v^2 + v^4/2\mathbf{c}^2 = = 2\,GM/r + B$ (constant) (5.19)

Let : $X = 2\,GM/r + B$

Let's rewrite equation (5.17) with this change of variable, limiting ourselves to order 2 in X

[because the ratio $(X^2/\mathbf{c}^2)/X$ is of the same order as $(\mathbf{v/c})^2$]:

(5.19) → $v^2 = X - X^2/2\,\mathbf{c}^2$

$$\Upsilon^2 v^2 = v^2 + v^4/\mathbf{c}^2 = X + X^2/2\mathbf{c}^2$$

On the other hand, equation (5.5) gives :

$$E_0 = E_{0\infty}\exp(GM/\mathbf{c}^2 r)$$

or: $m_0 = m_\infty (1 + GM/ \mathbf{c}^2 r) = m_\infty (1 + (X - B)/2\mathbf{c}^2)$

[we limit ourselves to the term in X because of the multiplication by $\Upsilon^2 v^2$ to be carried out afterwards]

$$m_0^2 = m_\infty^2 (K + X /\mathbf{c}^2)$$

$$m_0^2 \Upsilon^2 v^2 = m_\infty^2 (K + X /\mathbf{c}^2) (X + X^2/2\mathbf{c}^2) = m_\infty^2 (K X + 3X^2/2\mathbf{c}^2)$$

$$(\Upsilon m_0 v/A)^2 - (1/r)^2 = (m_\infty/A)^2 (K X + 3X^2/2\mathbf{c}^2) - (X-B)^2/(2GM)^2 \qquad (5.20)$$

The constant A can be calculated from the distance to the sun and the speed at perihelion or aphelion, where the velocity vector is perpendicular to the radius vector:

$$\psi = \pi/2 \quad \text{so} \quad \sin\psi = 1 \quad \text{and} \quad A = \Upsilon_i m_i v_i r_i$$

$$m_\infty/A = m_\infty/(\Upsilon_i m_i v_i r_i) \approx 1/(v_i r_i)$$

The second member of (5.20) can be written as a second degree polynomial in X :

$$(\Upsilon m_0 v/A)^2 - (1/r)^2 = -(1/(2GM)^2 - 3/(2\mathbf{c}^2 v_i^2 r_i^2))X^2 + P X + Q$$

There are two constants C and D such that, with the change of variable Y = X - C, we can write :

$$(\Upsilon m_0 v/A)^2 - (1/r)^2 = (1/(2GM)^2 - 3/(2\mathbf{c}^2 v_i^2 r_i^2))(D^2 - Y^2)$$

$$= (1/(2GM)^2)(1 - 6 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2))(D^2 - Y^2)$$

Furthermore: $1/r = (X - B)/2GM \rightarrow d(1/r) = dX/2GM = dY/2GM$

Given the two expressions above, equation (5.17) becomes :

$$\varphi = -\int dY/(1 - 6 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2))^{1/2}/(D^2 - Y^2)^{1/2}$$

$6 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2)$ is very small compared with 1, so we can write :

$$\varphi = -(1 + 3 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2)) \int dY/(D^2 - Y^2)^{1/2}$$

Therefore : $$\varphi = -(1 + 3 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2)) \arcsin (Y/D) + \text{Constant} \qquad (5.21)$$

5.2.3. We have seen that the passage to perihelion and aphelion is characterised by : $\Upsilon_i m_i v_i r_i = A$,

therefore: $(\Upsilon_i m_i v_i/A)^2 - (1/r_i)^2 = 0$

The result is: $D^2 - Y^2 = 0$, or : $\begin{cases} Y/D = 1 \text{ au périhélie} \\ Y/D = -1 \text{ à l'aphélie} \end{cases}$

The variation in φ between perihelion and aphelion is therefore equal to:

$$\varphi_a - \varphi_p = -(1 + 3 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2)) (\arcsin(-1) - \arcsin(1)) = (1 + 3 (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2)) \pi$$

For one revolution, the offset is (with v_i and r_i at perihelion or aphelion):

$$\Delta\varphi = 6 \pi (GM)^2/(\mathbf{c}^2 v_i^2 r_i^2) \qquad (5.22)$$

This formula is equivalent to that given by General Relativity (a = semi-major axis, e = eccentricity):

(GR) $$\Delta\varphi = 6\,\pi\,GM/\,\mathbf{c}^2 a(1-e)^2 \qquad (5.23)$$

Application to Mercury

$GM = 132\,712\,440\,018\ km^3\,s^{-2}$

$\mathbf{c} = 299\,792\ km\,s^{-1}$

$v_i = 58{,}98\ km\,s^{-1}$ (perihelion)

$r_i = 46\ 10^6\ km$ (perihelion)

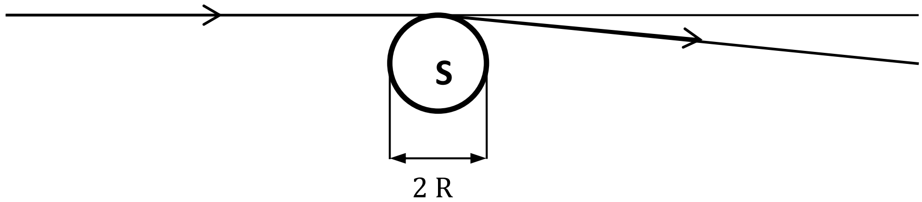
$\Delta\varphi = 5.018\ 10\text{-}7$ radians

$= 0.1035$ seconds

For one century (415 revolutions) :

$\boxed{\Delta\varphi = 42.95 \text{ arc seconds}}$

5.3. Curvature of light rays



Consider the path of a photon in the vicinity of the Sun, shown in the figure above.

By replacing the momentum $\Upsilon m_0\,v$ by $E/\mathbf{c}$ for the photon, equation (5.17) is written :

$$\varphi = -\int d(1/r)/((E/A\mathbf{c})^2 - (1/r)^2)^{1/2} \qquad (5.24)$$

This assumes that the energy of the photon is affected by the gravitational field in the same way as the total energy of a massive particle.

We check that $GM/\mathbf{c}^2 r$ *is small compared to* 1 *(equal to* $2.12\ 10^{-6}$ *for* $r = R$*)*.

Equation (5.6) is therefore valid:

$$E = E_\infty \exp(2\,GM/\,\mathbf{c}^2 r)$$

According to (5.16') : $E/A\mathbf{c} = 1/\,r \sin\psi$

For r = R (radius of the sun), $\Psi = \pi/2$, $\sin\Psi = 1$

So : $E_R/A\mathbf{c} = E_\infty \exp(2\,GM/\,\mathbf{c}^2 R) = 1/\,R$

$$E_\infty/A\mathbf{c} = (1/R)\exp(-2\,GM/\,\mathbf{c}^2 R)$$

$$E/A\mathbf{c} = (1/R)\exp(2GM(1/r - 1/R)/\mathbf{c}^2) = (1/R)(1 + 2GM(1/r - 1/R)/\mathbf{c}^2)$$

(5.24) → $$\varphi = -\int d(1/r)/((1/R)^2(1 + 4GM(1/r - 1/R)/\mathbf{c}^2) - (1/r)^2)^{1/2}$$

$$\varphi = -\int d(1/r)/((1/R)^2(1 - 2GM/\mathbf{c}^2R)^2 - (1/r - 2GM/\mathbf{c}^2R^2)^2)^{1/2}$$

$$\varphi = -\int d(1/r)/((1/R - 2GM/\mathbf{c}^2R^2)^2 - (1/r - 2GM/\mathbf{c}^2R^2)^2)^{1/2}$$

Integrating with respect to $(1/r - 2GM/\mathbf{c}^2R^2)$ gives :

$$\varphi = \arcsin((1/r - 2GM/\mathbf{c}^2R^2)/(1/R - 2GM/\mathbf{c}^2R^2)) + \text{Constant.}$$

For r varying from $+\infty$ to R the variation of φ is :

$$\varphi_R - \varphi_\infty = \arcsin 1 - \arcsin(-2GM/\mathbf{c}^2R)$$

$$= \pi/2 + 2GM/\mathbf{c}^2R$$

At the point closest to the centre of the sun, the angle of deviation of the trajectory is $(2GM/\mathbf{c}^2R)$; this must be doubled to obtain the total deviation:

$$\Delta\varphi = 4GM/\mathbf{c}^2R \qquad (5.25)$$

This is the value provided by the theory of General Relativity and verified by experiment:

$\Delta\varphi$ = 1.75 arc seconds

5.4. Pound and Rebka experiment (spectral gravitational effect)

Remember: *we are in the case where* $GM/\mathbf{c}^2r$ *is small compared to 1.*

We will show that the relation (5.14), giving the gravitational spectral shift, remains compatible with the results of the Pound and Rebka experiment if this experiment is interpreted within the framework of the new approach we are proposing.

The principle behind this experiment is to detect the effect of the Earth's gravity on photons:

Two samples of iron ^{57}Fe (emitter of gamma photons) are placed in a tower at a vertical distance sufficient to create a detectable gravitational effect (22.5 metres). By de-excitation, the source sample emits photons in a very narrow energy band; these photons are absorbed by the other sample if their frequency has remained very close to their emission frequency (excitation at the resonance frequency).

Because of the Mössbauer effect, the emission or absorption of a photon disturbs the variation in the frequency of this photon, due to the gravitational effect, to an extent sufficiently small that it remains detectable in the following way: by setting the transmitter in motion, we create a Doppler effect which counterbalances the gravitational frequency variation so as to allow the effective absorption of the photons. All we need to do is measure the Doppler frequency shift.

Let's consider the case of the transmitter placed on the ground. Let us denote by :

ν_S the frequency of the photons emitted,

ν_h the resonant frequency at the top of the tower (height h)

ν_S' the frequency of the photons arriving at the receiver,

v the relative velocity between transmitter and receiver.

Because of the low relative velocity, the energy variation factors linked to the ϒ parameter (of order $v^2/\mathbf{c}^2$ *whereas the Doppler effect is of order* $v/\mathbf{c}$*) are neglected.*

Since the receiver is offset by the height h from the transmitter, the energy of the atoms in the two devices is shifted in accordance with equation (5.5) for rest energies; the same applies to the energy and therefore the frequency of the emitted and resonant photons. Equation (5.15) must therefore be used:

$$\nu_2 = \nu_1 \ (1 + GM/\mathbf{c}^2 r_2)/(1 + GM/\mathbf{c}^2 r_1) \qquad (5.26)$$

which can be written as : $\nu_2 = \nu_1 (1 + (GM/\mathbf{c}^2)(1/r_2 - 1/r_1))$

In this case: $r_2 = r_1 + h$, so: $1/r_2 - 1/r_1 = - h/ r_1 (r_1 + h) \approx - h/r_1^2$

The relationship between the frequencies of the emitted and resonant photons is :

$$\nu_h = \nu_S (1 - GM\, h/\mathbf{c}^2 r_1^2)$$

Excluding the Doppler effect, the frequency shift of the emitted photon is given by equation (5.14), which, using the same reasoning as above, leads to :

$$\nu_S' = \nu_S (1 - 2GM\, h/\mathbf{c}^2 r_1^2)$$

Therefore: $\nu_S'/\nu_h = (1 - GM\, h/\mathbf{c}^2 r_1^2)$

The Doppler compensation must be such that :

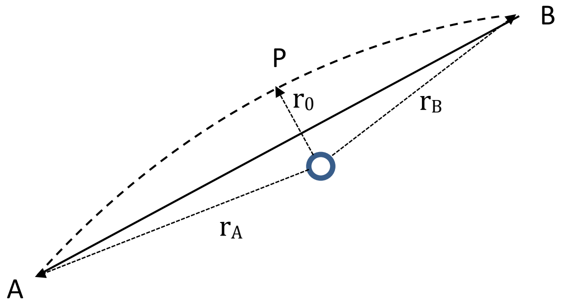
$$(1 + v/\mathbf{c})\, \nu_S'/\nu_h = 1$$

which leads to : $v/\mathbf{c} = GM\, h/\mathbf{c}^2 r_1^2$

$v = g\, h/\mathbf{c}$ which is the result of the experiment.

5.5. Shapiro effect

Let's take a look at the travel time of an electromagnetic signal between two points A and B, depending on whether or not this travel is influenced by the presence of a gravitational field.



We design by :

- r_A : the radial coordinate of A,
- r_B : the radial coordinate of B,
- r_0 : the radial coordinate at the point P on the trajectory closest to the source of the gravitational field.

We assume that $\mathbf{GM/c^2}\, r$ *is small compared to* 1 *for any value of r.*

We have seen (cf. § 5.1.3) that the speed of an electromagnetic wave is modified by the presence of the gravitational field. Its speed is given by equation (5.13) :

$$c_g = (1 - (2\ GM/\ \mathbf{c}^2 r))\ c$$

We can write: $c_g = \mathbf{c}\ (1 - \Delta\mathbf{c}/\mathbf{c})$

with : $\Delta\mathbf{c}/\mathbf{c} = (2\ GM/\mathbf{c}^2 r)$ (5.27)

To calculate the journey time, let's go back to the information given at the beginning of paragraph 5.2.1.

$dr/dt = c_g \cos\psi \;\rightarrow\; t = \int (1\ /c_g)\ dr/\cos\psi$ (5.28)

$$t = (1/\mathbf{c})\int(1 + \Delta\mathbf{c}/\mathbf{c})\ dr/\cos\Psi \quad (5.28')$$

The journey time is made up of two terms:

- the first [$t = (1/\mathbf{c})\int dr/\cos\Psi$] corresponding to the travel time at speed **c** ;
- the second [$\Delta t' = (1/\mathbf{c})\int(\Delta\mathbf{c}/\mathbf{c})\ dr/\cos\Psi$] representing the additional time due to the decrease in the speed of light.

Term corresponding to the variation in distance :

Since gravitation bends the trajectory of light rays, we expect the travel time between two points to increase compared with what it would be in the absence of a gravitational field (straight line travel).

Referring back to the beginning of sub-chapter 5.3, we see that equations (5.6) and (5.16') allow us to express $\sin\Psi$ in the form :

$$\sin \Psi = (r_0 / r) \exp (2 GM (1/ r_0 - 1/r)/ \mathbf{c}^2$$

Let: $K = 2 GM/ \mathbf{c}^2 r_0$ et $X = r_0/r$

$$\sin \Psi = X (1 + K (1 - X))$$

$$\cos \Psi = (1 - X^2)^{1/2} (1 - 2 K X^2/(1 + X))$$

$$1/ \cos \Psi = (1 - X^2)^{-1/2} (1 + K X^2/(1 + X))$$

Since: $dr = (-r_0 /X^2) dX$

$[t = (1/\mathbf{c})\int dr/\cos \Psi]$ $\rightarrow$ $t = (1/\mathbf{c}) \int (1 - X^2)^{-1/2} dr - (K r_0/\mathbf{c}) \int (1 - X^2)^{-1/2}/(1 + X) dX$ (5.29)

The straight line travel time at speed **c** is given by:

$$t_0 = (1/\mathbf{c}) \int r (r^2 - r_0^2)^{-1/2} dr = (1/c) \int (1 - X^2)^{-1/2} dr$$

and the increase in duration by:

$$\Delta t = - (K r_0/\mathbf{c}) \int (1 - X^2)^{-1/2}/(1 + X) dX$$

By changing the variable $X = 1/ch Y$, we obtain :

$$\Delta t = 2 GM/\mathbf{c}^3 \int d Y / (1 + ch Y)$$

By integrating, we get : $\Delta t = (2 GM/\mathbf{c}^3) (ch Y - 1)/ sh Y$

or: $\Delta t = (2 GM/\mathbf{c}^3) (e^Y -1)/(e^Y +1)$ (5.30)

From A to B, the variation in duration is the sum of the variations from A to P and from B to P.

If r_A and r_B *are sufficiently larger than* r_0 , we can assume that at points A and B:

$$(e^Y -1)/(e^Y +1) \approx 1$$

At point P, we have: $ch Y = 1$, $Y =0$ and $(e^Y -1)/(e^Y +1) = 0$

Finally: $\Delta t_{AB} = 4 GM/\mathbf{c}^3$ (5.31)

Term corresponding to speed variation:

Referring to equations (5.28') and (5.29) we see that we can write :

$$\Delta t' = (1/c) \int (1 - X^2)^{-1/2} (\Delta \mathbf{c}/\mathbf{c}) dr$$

(5.27) $\rightarrow$ $\Delta t' = -(2GM/\mathbf{c}^3)\int (1-X^2)^{-1/2} X^{-1} dX$ (lower order terms are neglected)

Assuming $X = 1/ch Y$, the above equation reduces to :

$$\Delta t' = (2GM/\mathbf{c}^3) \int dY$$

By integrating : $\Delta t' = (2GM/\mathbf{c}^3) \arg ch(r/r_0)$ (5.32)

If r_A and r_B are sufficiently larger than r_0 , then: $\arg ch(r_A/r_0) \approx \ln(2r_A/r_0)$

$\arg ch(r_B/r_0) \approx \ln(2r_B/r_0)$

Since $\arg ch(1) = 0$: $\Delta t'_{AB} = (2GM/\mathbf{c}^3) \ln(4r_A r_B / r_0^2)$ (5.33)

Finally, summing equations (5.31) and (5.33) gives the full Shapiro effect (for a one-way AB):

$$\Delta T_{AB} = (2\,GM/\mathbf{c}^3)(\ln(4\,r_A r_B/r_0^2) + 2) \quad (5.34)$$

This effect is not exactly identical to that given by General Relativity :

(GR) $$\Delta T_{AB} = (2\,GM/\mathbf{c}^3)(\ln(4\,r_A r_B/r_0^2) + 1) \quad (5.35)$$

The secondary term corresponding to the variation in distance is doubled in (5.34) compared with (5.35). It should be noted that, under the conditions of the experiments carried out, this secondary term is too small compared with the main term to be detected.

We will explain this discrepancy in sub-chapter 5.6 below.

5.6. Comparison with the theory of general relativity

We have just examined the main experiments considered as a basis for verifying the theory of general relativity. We have seen that, *under weak gravitational field conditions*, the predictions of GR and those made using our new approach are identical except for the gravitational spectral shift and the secondary term of the Shapiro effect.

This similarity is explained below.

5.6.1. Schwarzschild metric

In the context of general relativity, the Schwarzschild metric is a solution of Einstein's equations. It describes the geometry of space-time deformed by the gravitational field outside an isolated, spherically symmetric, static (non-rotating), uncharged body surrounded by vacuum.

For a body of mass M, this metric is defined outside a sphere of radius :

$$R_S = 2\,GM/\mathbf{c}^2 \quad (5.37)$$

Using the spherical coordinates ($\mathbf{c}t$, r, θ, φ), the infinitesimal distance ds related to the metric is given by the relation :

$$ds^2 = (1 - R_S/r)\, \mathbf{c}^2\, dt^2 - (1 - R_S/r)^{-1}\, dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (5.38)$$

In distorted space-time, a particle subject only to the action of gravitation follows an inertial motion and moves along a geodesic line:

- of the "light" type, characterised by $ds^2 = 0$, for a particle of zero mass (photons) ;
- of the "time" type, characterised by $ds^2 = \mathbf{c}^2\, d\tau^2$, for a particle of non-zero mass ($d\tau$ being the proper time interval).

The study of motion is based on the conservation, along geodesics, of quantities assimilated to energy and angular momentum [34] .

Plane trajectory of a material particle (orbit of a planet)

The frame of reference is chosen such that the trajectory lies in the plane $\theta = \pi/2$: [35]

- the first quantity conserved is similar to the energy per unit mass outside the gravitational attraction (at infinity) [36] :

$$\varepsilon = \mathbf{c}^2\,(1 - R_S/r)\,(dt/d\tau) \qquad (5.39)$$

- the second is the angular momentum per unit mass (at infinity):

$$l = r\,(r\, d\varphi/dt)\,(dt/d\tau) = r\, v \sin\psi\,(dt/d\tau) \qquad (5.40)$$

Let's calculate $(dt/d\tau)$ from equation (5.38) and relations :

$$dr/dt = v\cos\psi \text{ and } d\varphi/dt = (v/r)\sin\psi$$

(5.38) → $d\tau^2 = ((1 - R_S/r) - (1 - R_S/r)^{-1}(v^2/\mathbf{c}^2)\cos^2\psi) - (v^2/\mathbf{c}^2)\sin^2\psi)\, dt^2$

$$d\tau^2 = ((1 - R_S/r) - (v^2/\mathbf{c}^2) - (R_S/r)(v^2/\mathbf{c}^2)\cos^2\psi)\, dt^2$$

$$d\tau^2 = ((1 - v^2/\mathbf{c}^2) - (R_S/r)(1 + (v^2/\mathbf{c}^2)\cos^2\psi))\, dt^2$$

With $\Upsilon^2 = 1/(1 - v^2/\mathbf{c}^2)$:

$$d\tau^2 = (1/\Upsilon^2)(1 - \Upsilon^2 (R_S/r)(1 + (v^2/\mathbf{c}^2)\cos^2\psi))\, dt^2$$

And finally: $dt/d\tau = \Upsilon\,(1 + \Upsilon^2 (R_S/r)(1 + (v^2/\mathbf{c}^2)\cos^2\psi)/2)$

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[34] This conservation can be deduced from the symmetries of space-time. See, for example, Eric Gourgoulhon's 2013-2014 General Relativity course (2ème year of the "Research, Astronomy and Astrophysics" master's degree at Paris Observatory and Universities of Paris 6, 7 and 11), available on the Internet.

[35] see figure § 5.1.4.

[36] Remember that GR considers rest mass to be invariant.

*R_S /r is small compared to 1. Let's also consider the non-relativistic case (v/**c** small compared to 1)*:

$$dt/d\tau \approx \Upsilon\,(1+ R_S /2r)$$

Relations (5.39) and (5.40) can be written as :

$$\varepsilon = \mathbf{c}^2\, \Upsilon\,(1- R_S /2r) \qquad (5.39')$$

$$l = \Upsilon\,(1- R_S /2r)\, r\, v \sin\psi \qquad (5.40')$$

The conservation condition for these quantities leads to the two equations below:

$$\Upsilon = \Upsilon_\infty\,(1- R_S /2r)^{-1} \qquad (5.41)$$

$$\Upsilon\,(1+ R_S /2r)\, r\, v \sin\psi = \text{A constant} \qquad (5.42)$$

Radial displacement of a photon (gravitational spectral shift)

Conservation of the energy quantity along the radial "light" geodesic followed by the photon gives :

$$E_{photon} = (1 - 2\,GM/\,\mathbf{c}^2 r)^{-1/2}\, E_\infty \qquad (5.43)$$

Furthermore, with ds = 0, dθ = 0 and dφ = 0, equation (5.38) allows us to express the (radial) speed of the photon in reference frame (**c**t, r, θ, φ) :

$$c_g = dr/dt = (1- R_S /r)\mathbf{c}$$

Non-radial trajectory of a photon (deflection of light rays and the Shapiro effect)

The trajectory of a photon remains in a plane. The calculations are based on non-radial "light" geodesics. The components of the photon's 4-impulse are deduced from the quantities conserved.

Using the metric then leads to the differential equation of the photon trajectory in the plane θ = π/2.

As with the radial trajectory, the photon speed can be determined from (5.38) :

$$c_g^2 /\mathbf{c}^2 = (1 - R_S /r)/(1 + (R_S /r) \cos^2\psi) = 1 - R_S /r - (R_S /r) \cos^2\psi$$

$$= 1 - 2\, R_S /r + (R_S /r) \sin^2\psi$$

$$c_g /\mathbf{c} = 1 - R /r_S + (R_S /r) \sin^2\psi/2 \qquad (5.44)$$

5.6.2. Explanation of similarities and differences

Fundamental differences between the two approaches

The first thing to remember is the following difference in concept:

- General relativity treats the problem kinematically. The curvature of space-time accelerates or slows down particles.[37] The energy of the source of the gravitational action defines the parameterisation of the metric. The energy variations of the particle subjected to this action are not explicit.
- The new approach proposed in this note focuses on energy with the principle of conservation of total energy. A real exchange of energy takes place between the particle and the gravitational field.[38] This approach is made possible by abandoning the principle of invariance of energy at rest.

The other fundamental difference is precisely the difference in the concept of rest energy. In weak field conditions, the total energy involved is doubled in the approach we are proposing compared with Newtonian gravitation.

Particle movement

The two approaches give identical predictions for the motion of particles placed in a gravitational field (*in weak field conditions*).

This is because :

- the Lorentz factor Υ is expressed in the same way: equations (5.11) and (5.41) are identical;
- the curvature of space-time[39] is equivalent to equating the value of the angular momentum of a particle to the one we propose: equations (5.16') and (5.42) are identical if we take into account the law giving the increase in rest energy in the gravitational field (5.5);
- As far as photons are concerned, both approaches assume that they are subject to gravitational action in the same way as material particles;
- both approaches lead to the same variation in photon speed in the gravitational field for trajectories close to radial trajectories.

[37] In the same way that a simple change of Galilean reference frame modifies the speed of a particle.
[38] This is explained in the note entitled "Gravitational Field, Fundamental Principle of Dynamics and Quantum Mechanics".
[39] With the Schwarzschild metric.

As far as the Shapiro effect is concerned, the difference noted in sub-chapter 5.5 is easily explained by equation (5.44):

This equation gives the complementary term $[+ (R_S /r) \sin^2 \psi /2]$ to equation (5.13) giving the speed of a photon in the gravitational field. If we complete the calculation in sub-chapter 5.5 (equation 5.27) taking into account the above term, we find a decrease in travel time equal to :

$$\Delta t'_1=(GM/\mathbf{c}^3)\int(1-X^2)^{-1/2}\, X\, dX \qquad \text{(taking into account that: } \sin\psi \approx X)$$

So, integrating: $\Delta t'_1= - (GM/\mathbf{c}^3)(1 -X^2)^{½}$

For a one-way AB trip: $\Delta t'_{1AB} = - 2GM/\mathbf{c}^3$

This difference is the same as that noted in sub-chapter 5.5.

Gravitational spectral shift

As explained in section 5.1.3, the gravitational spectral shift given by the GR is twice as small as that calculated using our approach.

This is a direct consequence of the fundamental difference in the concept of energy mentioned at the beginning of this paragraph. Since frequency is directly linked to energy by the Planck-Einstein relation, velocity does not come into play and curvature does not come into play to correct the discrepancy.

With regard to the Pound and Rebka experiment, the predictions of the two approaches are identical because the difference in total energy is compensated for by taking into account in our approach the difference in rest energies between transmitter and receiver (leading to the same difference in frequencies).[40]

[40] In paragraph 4.1.3 (Doppler effect), we already considered that the energy of photons emitted by a source varies as the energy of the source varies.